

Store Separation: Theoretical Investigation of Wing Aeroelastic Response

Giovanni Paolo Reina*, Angelo Della Sala*, Marco Evangelos Biancolini**, Corrado Groth**, Domenico Caridi†

*University of Naples “Federico II”, DICMaPI
Piazzale Tecchio 80, 80125 Napoli, Italia
gianpaolo.reina@libero.it, angelodellasala84@gmail.com

** University of Rome “Tor Vergata” Department of Enterprise Engineering, Via Politecnico 1, 00133 Rome, Italy. biancolini@ing.uniroma2.it

† ANSYS Italia S.r.l., Largo Augusto 7, 20122 Milano, Italy
domenico.caridi@ansys.com

Abstract

The objective of this paper is to propose a novel approach to the study of the steady and unsteady aeroelastic behavior of a wing during the flight, as for example in cases where there is a release of underwing bodies in military aircrafts. This new methodology is based on a structural modal superposition approach and a mesh morpher fully integrated in the fluid dynamic (CFD) solver, that allows to greatly reduce the analysis time of the classic approach to a FSI problem, typically characterized by an exchange of data between the structural and the fluid dynamic solver.

The following describes the whole procedure developed and the main obtained results.

Nomenclature

b	Wingspan, m
c	Chord, m
CAD	Computer Aided Design
CFD	Computational Fluid Dynamic
C_D	Drag Coefficient
C_L	Lift Coefficient
C_M	Moment Coefficient
C_p	Pressure Coefficient
FEA	Finite Element Analysis
FEM	Finite Element Method
FSI	Fluid Structure Interaction
H	Altitude, ft
MDM	Moving Deforming Mesh
α	Angle of attack, deg

1. Introduction

The structure of a wing, as is known, is not rigid, but all loads acting on it (wing weight, external stores weight, aerodynamic loads) cause its deformation. A representative example is given by military aircrafts at the moment in which one of the carried bodies is released and a sudden change of the total load occurs with a subsequent variation of its shape over the time. This is an unsteady event and the prediction of the transient behavior is capital because the pilot can be provided in advance with all information necessary to get the right reaction.

One strategy commonly adopted by the majority of multi-physics software allows to face the Fluid Structure Interaction (FSI) problem through an exchange of data between the structural solver and the fluid dynamic one. However, this involves a substantial overhead in the analysis times, since it is necessary to modify the geometry and the mesh consistently with the deformations calculated by the FEM module external to the CFD solver at each time step.

The methodology presented in this paper, is based on the use of the FSI module of RBF Morph, a morpher based on Radial Basis Functions and fully integrated in ANSYS Fluent. Thanks to structural modes embedding, the calculation time overhead due to structural deformations is very small, if compared with the rigid case, and is mainly due to the enabling of Fluent Moving and Deforming Mesh (MDM) algorithm (M.E. Biancolini, 2013) as the RBF information use for updating mesh position is pre-computed at the beginning of the process and just a simple linear combination of retained modes is required at each time step.

This goal was achieved through a specific workflow, which will be explained in detail in the following sections. The first step was the construction of a CAD relative to a wing and an underwing body, neglecting the connection element, and the generation of computational grids. Subsequently the vibration modes of the wing structure (assumed as a homogeneous bulky component) were obtained and structural analysis was performed to determine the effect of the loads acting on the wing. Then, using the obtained modal displacement and nodal forces were performed two different FSI analyses: an aeroelastic steady analysis to determine the condition of elastic equilibrium of the wing in the presence of the store and, starting from this, an unsteady analysis to get the transient aeroelastic response of the wing due to the release of the underwing body. This event was simulated by imposing the disappearance of the store.

In this way it is possible to set an unsteady FSI analysis updating the CFD mesh at each time step according to the aerodynamic and inertial loads, allowing to carry all the calculations (fluid and structure) within the CFD solver without having to periodically exchange data such as geometry and mesh. The modal superposition method exploited in this paper is valid under the assumption of linear deformations that for the aeroelastic response of the wing considered in this study was assumed acceptable

2. Theory background

The system of differential equations that govern the motion of a system with n degrees of freedom, characterized by the stiffness $[K]$ and by the damping $[C]$ and subjected to the forcing $F(t)$ can be expressed in compact form as follows:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \quad (2.1)$$

Assuming that there are no viscous damping and external forcing, the equation becomes:

$$[M]\{\ddot{x}\} + [K]\{x\} = \mathbf{0} \quad (2.2)$$

In this case we can assume that the response of the system is described by harmonic functions of the type:

$$\{x\} = \{X\}\sin(\omega_n t - \phi) \quad (2.3)$$

with X and ϕ defined by appropriate boundary conditions and ω_n equal to:

$$\omega_n = \sqrt{K/M} \quad (2.4)$$

Suitably manipulating Eq. 2.2 it can be derived the following expression:

$$([K] - \omega_i^2[M])\{X\} = \mathbf{0} \quad (2.5)$$

As it is known, the modal analysis is the science that studies the dynamic behavior of a structure in relation to the forces that excite it. Through this science it is possible to calculate the natural vibration modes of a structure (eigenvectors) and their respective frequencies (eigenvalues). This is done by solving the secular equation given by Eq. 2.5 in the unknowns ω^2 . Calculated natural frequencies and natural modes of oscillation are representative of the system response at the resonance conditions.

As it is demonstrated that the eigenvectors possess the property of orthogonality, it is possible to introduce a base change from normal basis to modal orthogonal one, so that the equations of motion can be uncoupled, thus as to bring only one unknown in each equation. The simplification that can be reached is evident as each equation can be solved separately from the others, also the contribution of each vibration mode to the total deformed would be to emerge.

To realize the change of basis we introduce the vector of modal coordinates $\{q\}$ defined as follows:

$$\{q\} = [X]^{-1}\{x\} \quad (2.6)$$

in which $[X]$ is the modal matrix whose columns are formed by all the eigenvectors normalized with respect to the mass:

$$[X] = [X_1|X_2| \dots |X_n] \quad (2.7)$$

It should be emphasized that must be true for the matrix of the masses the following condition:

$$[X]^T[M][X] = [I] \quad (2.8)$$

while for the stiffness matrix:

$$[\Omega] = [X]^T[M][X] = \begin{bmatrix} \omega_1^2 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \omega_2^2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \omega_n^2 \end{bmatrix} \quad (2.9)$$

known as the spectral matrix, whose main diagonal the eigenvalues are willing and having the remaining elements equal to zero. At this point, replacing the 2.6 in 2.1 and premultiplying each term for $[X]^T$ is obtained:

$$[I]\{\ddot{q}\} + [X]^T[C][X]\{\dot{q}\} + [\Omega]\{q\} = [X]^T\{F(t)\} \quad (2.10)$$

To be able to finally uncouple the equations we must suppose that the matrix $[X]$ also makes diagonal the matrix of damping $[C]$. This assumption is valid in the case in which $[C]$ can be expressed as follows:

$$[C] = (a[M] + b[K]) \quad (2.11)$$

i.e. it must be a linear combination of the mass and stiffness matrices. This result is not rigorous only in the event that there are devices that act directly and explicitly on $[C]$. In all other cases, it is reasonable to assume that the viscous damping matrix is diagonalized by the modal matrix. In this way each equation of 2.10 can be considered decoupled from the other and can be posed in the form:

$$M_{ii}\ddot{q}_i + C_{ii}\dot{q}_i + K_{ii}q_i = F_i(t) \quad (2.12)$$

For convenience, the above equation is often presented in the following equivalent form:

$$\ddot{q}_i + 2\zeta_i\omega_i\dot{q}_i + \omega_i^2q_i = \frac{F_i(t)}{M_{ii}} \quad (2.13)$$

in which with the letters ω_i and ζ_i it is indicated, respectively, the natural frequency of the system and the damping factor, defined by the following expressions:

$$\omega_i = \sqrt{\frac{K_{ii}}{M_{ii}}} \quad \zeta_i = \frac{C_{ii}}{2M\omega_i} \quad (2.14)$$

Finally, we conclude the discussion of the theory of modal focusing on how to get the deformed structure in normal coordinates but using the modal basis. In fact for the Eq. 2.6 we can express the deformed of a structure, in normal coordinates, as follows:

$$\{x\} = [X]\{q\} \quad (2.15)$$

where the columns of the matrix $[X]$ are precisely the natural modes of the structure. In other words, the deformed shape of a system is expressed as the combination of the vibration modes of the same one weighed through a participation factor q . This factor must be obtained from the solution of Eq. 2.13 (Meirovitch, 2010).

3. Problem description and workflow

The store separation problem is particularly felt on high-performance military attack aircraft, that typically carry vast arrays of air-to-air or air-to-ground weapons on wing-mounted pylons. The substantial weight and aerodynamic characteristics of these external stores can significantly change the aeroelastic behaviour and structural response of the wings. (Chambers, 2005)

Assuming an airplane in flight at a certain subsonic Mach number, the wings will be in a condition of elastic equilibrium, due to the different loads applied (wing weight, external stores weight, aerodynamic loads). If at a certain moment there is the release of one of the external stores, the wing will lose its equilibrium beginning to oscillate. The purpose of this work is to precisely calculate this wing aeroelastic response following an event of store separation.

The whole analysis hereby presented was carried within the ANSYS Workbench environment. ANSYS Workbench is particularly useful since it integrates simulation tools for various disciplines, allowing for instance to establish multi-disciplinary simulation flows, to use pre-post processing tools unified, to build complex coupled analyses involving multiple physics and to manage parametric analysis. The development of the workflow in Workbench is shown in Figure 1.

The mesh morpher used was RBF Morph, a morpher fully integrated in the CFD solving process. RBF Morph uses a series of radial basis functions (RBFs) to produce a solution for the mesh movement using

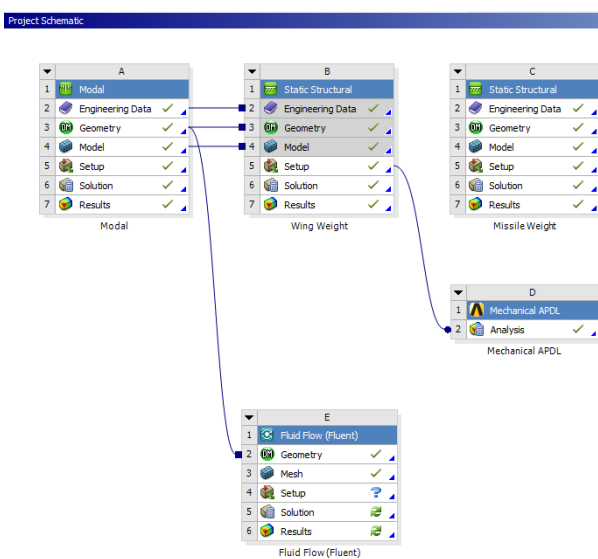


Figure 1 Workbench project schematic

source point defined by the user and their displacements. The solution of the RBF problem, calculated for the source points, can be stored if needed and later applied to the volume mesh by using a smoother. This morphing operation can be executed in a matter of seconds, even on very large meshes, by using the parallel processing capabilities of high performance computing (HPC) clusters.

Since the aim of this work is to determine a methodology, we have chosen to refer to suitably simplified geometries. For this reason we have created a geometry from a known airfoil and shaped the store as a body of revolution, neglecting any connecting element between the two bodies. In particular the CAD of the wing was obtained from the simple extrusion of the known NACA 0012 airfoil, shown in Figure 2. With reference to several military aircrafts, the half wingspan was imposed to 2.5 m.

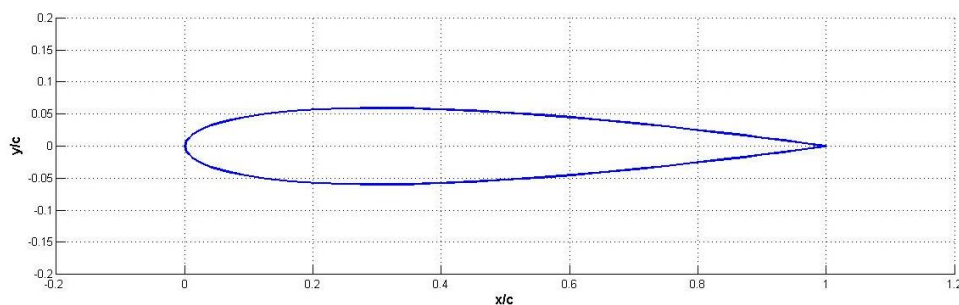


Figure 2 NACA 0012 Airfoil

Regarding the store, the goal was to create a simple model of an air-to-ground missile, that generally weights more than 650 lb. The modelled geometry was generated by joining a cylinder and a hemisphere. The final geometry can be seen in Figure 3.

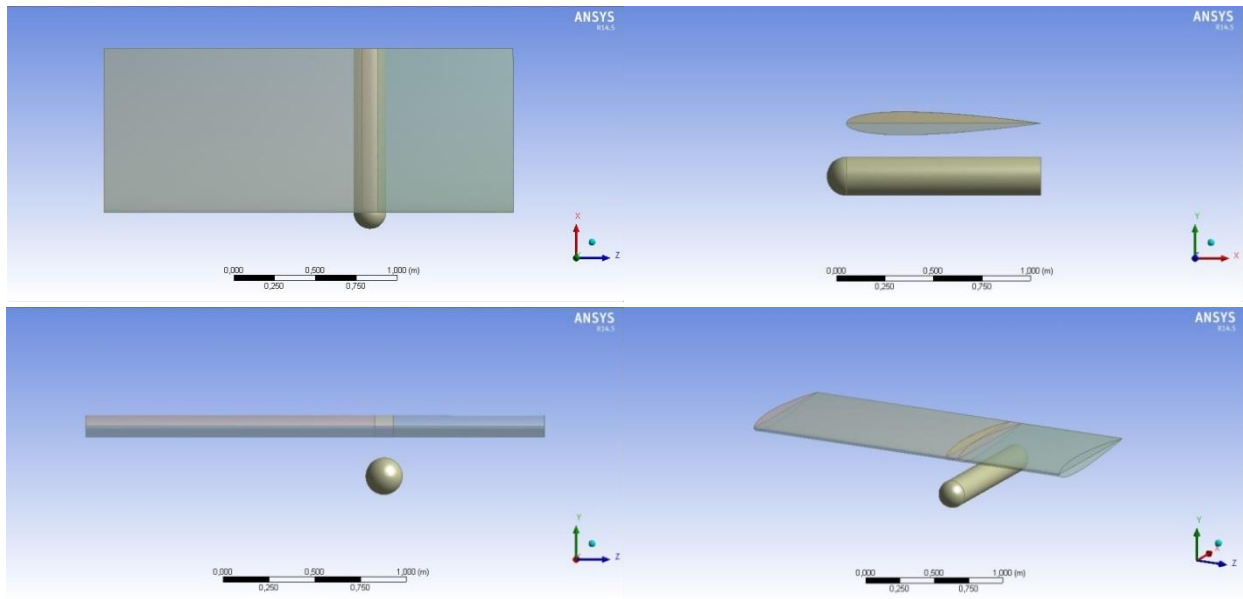


Figure 3 Orthographic projection and isometric view of the model

After the creation of the geometry the CFD and FEM meshes were generated. For the realization of the first one, the guidelines adopted in the choice of the various parameters are essentially two: keeping the number of elements on the order of magnitude of one million and obtaining a mesh of acceptable quality. Such a workflow is intentionally simplified and is intended just as a demonstrator of the approach to be used for high fidelity CFD.

Based on these objectives, a coarse unstructured CFD mesh was generated throughout the fluid domain but the area close to the wing surface, where ten layers of hexahedral cells were inflated in order to better calculate the boundary layer. Finally a tetrahedral mesh conformal to the outside one was generated inside the store: this grid allows to simulate the fall of the missile with its disappearance, i.e. transforming the wall in an internal face. In Figures 4 is shown a detail of the 740000 cells mesh with a maximum skewness of 0.914, while in Figure 5 is highlighted the inflation on the wing.

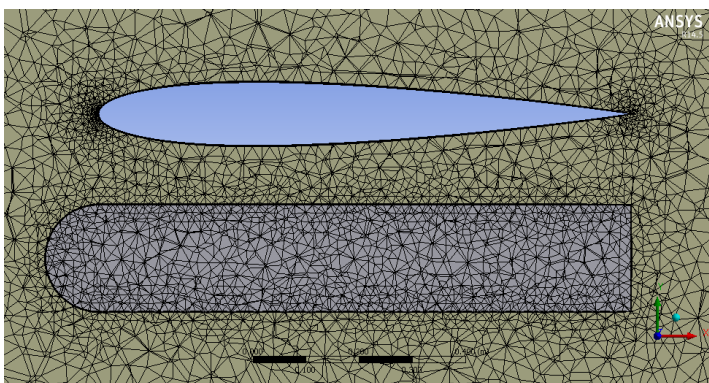


Figure 4 Detail of the mesh for CFD Analysis

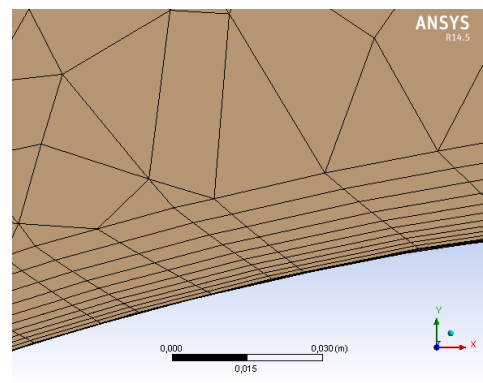


Figure 5 Inflation

The structural mesh of the wing is comprised of 1260 hexahedral solid elements and 6930 nodes. It is shown in Figure 6.

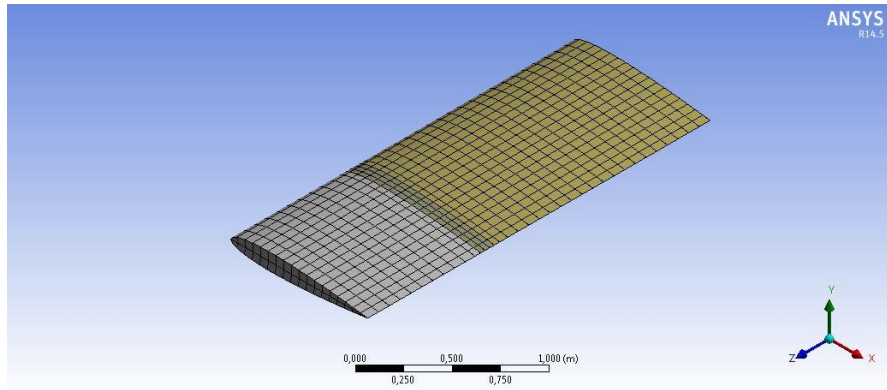


Figure 6 Structural Mesh

Once the structural mesh was obtained the modal and structural analyses of the wing were carried out. A linear elastic material has been selected (Al6061-T6); the model has been constrained fixing all the nodes on the surface at the root of the airfoil. The first six modes of vibration and the relative frequencies were calculated (see Figure 7).

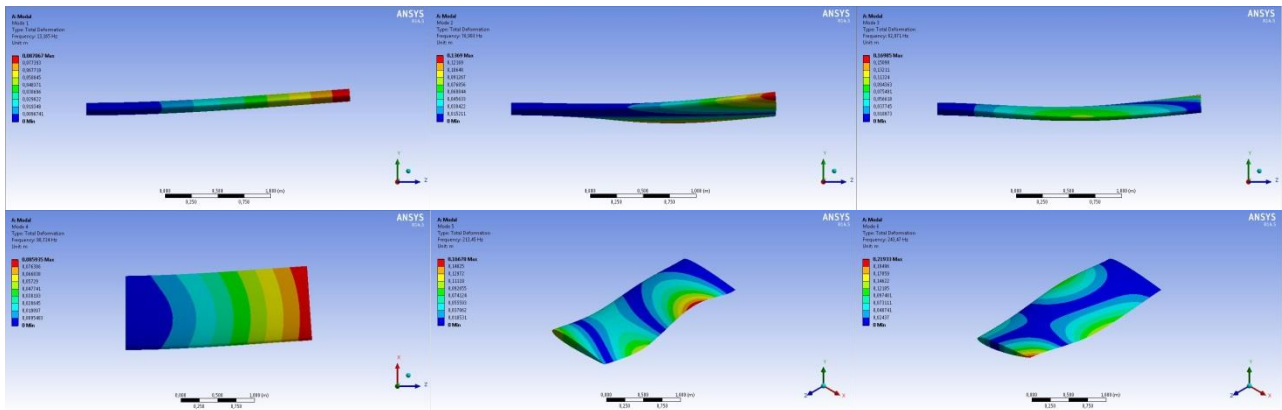


Figure 7 Modal analysis results

The same structural model used for modal analysis was used also for the determination of the effect of the loads acting on the wing. In particular, were considered two loads: the weight of the structure itself and the weight of the store, supposed to be applied only to the wing sections which in reality are in contact with the pylon. In order to emphasize elastic effects even with a stiff bulky wing assembly, it was decided to consider a hundredfold heavier store than the reality.

Considering only the weight of the wing, the tip has shown a deflection equal to 0.002 m, while the presence of the store has caused a deflection equal to 0.145 m.

3.1 Calculation of modal solutions and modal forces in RBF Morph

At this point, in order to proceed to the CFD simulations, it was necessary to import the results obtained from the structural analyses in ANSYS Fluent. The vibration modes of the wing were exported as a points cloud with known displacements and used as input data for the mesh morphing tool RBF Morph (each FEM node is exported as an RBF points' cloud), obtaining an RBF solution for every modal shape (Biancolini, October 2011). In Figure 8 are shown the selected points highlighted in the Fluent graphic window.

After obtaining the six solutions the six amplification factors corresponding to the two static loads acting on the wing were calculated, i.e. the weight of the store and the self weight of the wing. In the first case, the determination of these coefficients was made directly with RBF Morph. In fact, knowing the weight of the store and deducing from ANSYS Mechanical the surface of the wing section on which it acts, it was possible to introduce this effect in ANSYS Fluent as a pressure value, transforming the same section in a *pressure outlet* and applying a pressure that produces exactly the desired resultant load. This is possible because RBF Morph FSI module comes with a functionality that allows to transform pressure loads acting on wetted surfaces in modal loads. Subsequently, once initialized the vibration modes with RBF Morph's FSI module using the command:

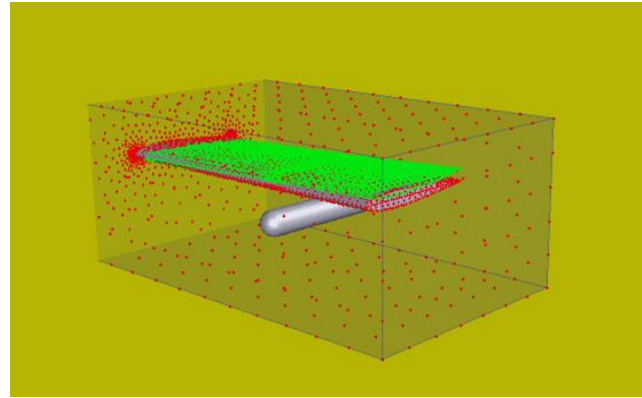


Figure 8 Selected Points in graphic window

```
(rbf-fmorph-init ("Mode1" 0) ("Mode2" 0) ("Mode3" 0) ("Mode4" 0) ("Mode5" 0) ("Mode6" 0))
```

it was possible to calculate the modal forces acting on the wing simply using the command

```
(rbf-fmorph-forces '(surface list))
```

in which surface list denotes the list of all the named selections of surfaces of interest, in our case the wing ones. The result is shown in Figure 9.

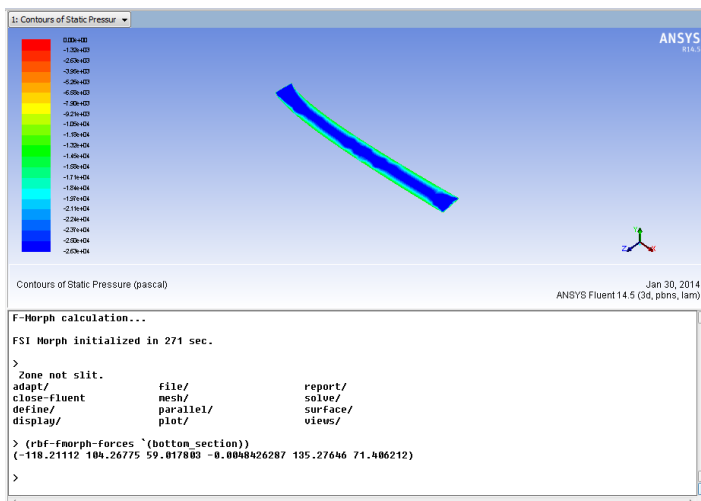


Figure 9 Calculation of the force due to the store weight on the wing

To obtain the amplification factors finally these forces were divided by the square of the respective six pulsations, derivable from the frequencies of the modes previously calculated.

Regarding the factors related to the weight of the wing structure, however, another procedure was followed. Once exported from mechanical all the nodal forces acting on the structure in the three directions x, y, and z, the six modal forces relative to the weight of the wing were obtained by doing the scalar product between them and the modal displacements

3.2 Steady FSI Analysis

As explained in the introduction, the aim of this work was to determine the aeroelastic transient following the separation of the store. Before this, however, it was necessary to calculate the elastic equilibrium condition of the wing in the presence of the store, since it represents the starting point for the subsequent unsteady calculation.

It is important to emphasize that at this stage a simplified approach was followed, not solving the equation of the full dynamic (Eq. 2.13), but simplifying it under the assumption of static loads. In the case in which the speed of deformation of the structure appears to be much faster than the speed of application of the loads, the phenomenon can be considered static and Eq. 2.13 can be reduced in the following form:

$$\omega_i^2 \mathbf{q}_i = \frac{F_i(t)}{M_{ii}} \quad (3.1)$$

Combining this equation with Eq. 2.15 we obtain the total deformation of the system, expressed in normal coordinates, by adding all the different contributions of the N modes of vibration:

$$\{\mathbf{x}\} = \sum_{i=1}^N \{\mathbf{X}_i\} \mathbf{q}_i = \sum_{i=1}^N \{\mathbf{X}_i\} \frac{F_i(t)}{M_{ii} \omega_i^2} \quad (3.2)$$

The terms within the summation of Eq. 3.2 thus represent the weights with which the vibration modes participate to the total deformation of the system expressed in normal basis. In other words, the deformation of the structure can then be expressed as a combination of different vibration modes weighted by the modal participation factor $\frac{F_i(t)}{M_{ii} \omega_i^2}$. A continuous system, however, has infinite degrees of freedom and infinite natural modes, namely infinite pulsations and endless mode shapes. Unlike modes, pulsations are distinct and constitute a discrete infinite, while the modes, while not constituting an infinite discrete, above certain frequencies no longer have physical sense and in any case are never excited. For this reason, the number of modes to be considered within the summation is limited to a finite number, six in our case.

The procedure followed for the steady analysis can be summarized in the following points:

- Loading of the solutions for the individual vibration modes within Fluent using the FSI module of RBF Morph;
- Setting a steady calculation (see Table 1 for the parameters of the test point);

Mach Number	0.29
Altitude	0 ft
Reynolds Number	7e6
α	0 deg

Table 1 Test point

- Implementation of a scheme file containing the FSI subroutine of RBF Morph to update the mesh;
- Setting the number of iterations between each mesh update by using the Fluent calculation activities;
- Start of the calculation.

The most interesting aspect of this analysis is definitely related to the implementation of the morpher during the fluid dynamic calculation. It in fact has the task to "upgrade" the mesh considering in addition to constant loads (wing weight and weight of the store) also the pressure one generated during the computation and due to the motion field, until reaching an equilibrium condition. Assuming that the phenomenon is static, the

deformed system can be expressed as a combination of a number of vibration modes weighted by the modal participation factor. This obviously depends on all the forcing agents on the structure, and therefore henceforth will necessarily also consider those related to the pressure field.

It is obvious, however, that the factors of participation due to the latter cannot be obtained from the outset as to the weights acting on the structure, but must be derived within the CFD software during the calculation of the solution. To do this the loads acting on wetted surfaces are taken into account and used to calculate the modal forces and the modal coordinates for each mode directly by RBF Morph during the calculation stage. To avoid introducing noise in the calculation this process and the mesh update are not accomplished every iteration but at convergence.

Following the procedure outlined it was proved that the effect of aerodynamic load tends to reduce the deflection of the wing tip of a percentage amount equal to -17% at presence of the store. Figure 10 shows the final wing configuration after the steady analysis.

3.3 Unsteady FSI Analysis

Once derived the condition of elastic equilibrium in the case of loaded wing, it was possible to proceed with the unsteady case to determine the transient from the disappearance of the missile. To do this a procedure very similar to the static case was followed, i.e. loading solutions for the individual modes of vibration, setting the unsteady calculation by appropriately choosing the size of the time step and finally updating the mesh at the end of each time step.

For this task the unsteady RBF Morph FSI module was used, removing the load of the store and imposing as an initial condition the deformed shape obtained by the steady analysis.

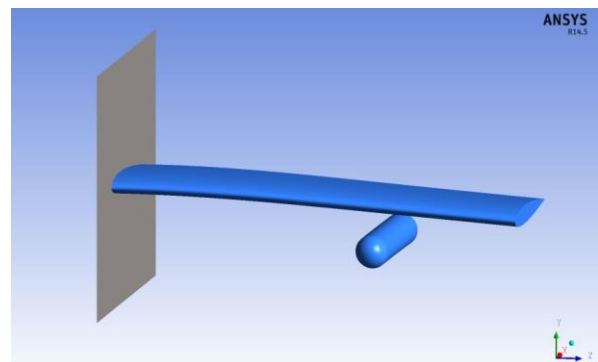


Figure 10 Final wing configuration after steady FSI calculation

From the point of view of the flow field, then, it was decided to simulate the fall of the store making it disappear at the beginning of the calculation, transforming its surfaces from *wall* to *interior* inside the *Boundary Conditions* panel and changing the internal material from solid to fluid in the *Cell Zone Conditions*. The unsteady calculation was performed with an initial time step size of $1e-4$ up to a maximum value of $1e-3$ and with a morpher intervention at the end of each time step.

One of the main results of this work is that this approach involves a moderate overhead of only 9% with respect to the rigid unsteady calculation without the mesh morpher, making the FSI approach affordable and attractive especially considering that data exchanging between CFD and FEA mesh at each time step is avoided (Cella U., 2012).

Another important result is definitely provided by the monitors of the aerodynamic coefficient. In particular, we are interested in evaluating the evolution of the lift coefficient: it is easy to imagine, in fact, that the release of such a heavy body influences the wing structure especially in the vertical direction. Figure 11 shows the trend of C_L obtained (blue line): it can be seen as the diagram is characterized by large fluctuations, related to oscillation of the wing after the release of the store, that tend to fade after

approximately 0.6 s. Furthermore it is evident that in the first instants occurs a significant increase of aerodynamic load, whose prediction is very important to evaluate the behavior of the wing from the structural point of view. This increase in load is even more significant when compared with the trend of C_L in the case of rigid wing, previously calculated (green line).

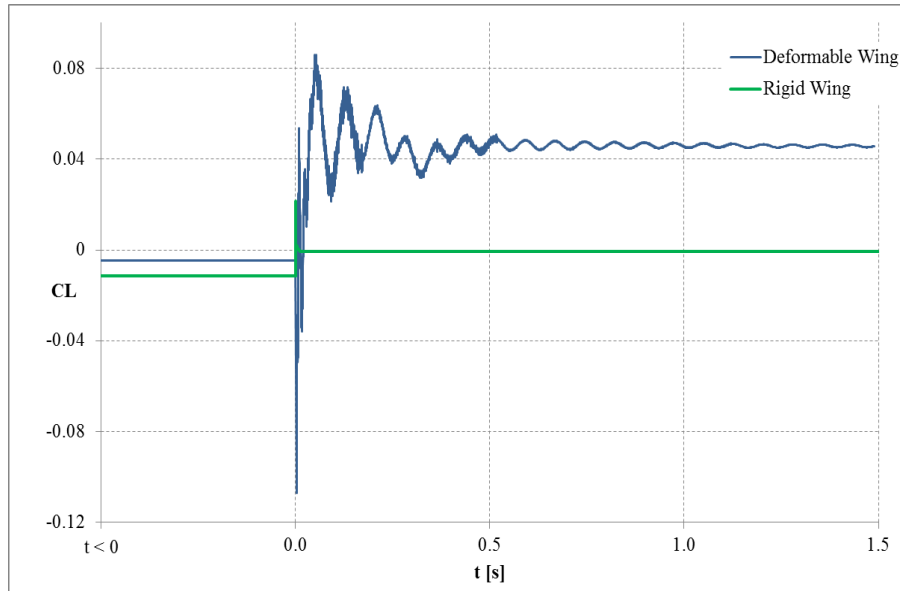


Figure 11 Lift Coefficient vs. time in cases Deformable Wing and Rigid Wing

Figure 12 shows instead a comparison between the trend of C_L and the graph of the dimensionless wing tip deflection $\frac{Y}{Y_0}$, in which Y_0 is the tip deflection in presence of the store, as a function of dimensionless time $\frac{\omega t}{2\pi}$. It is clearly noted that the damped oscillatory is similar for both curves.

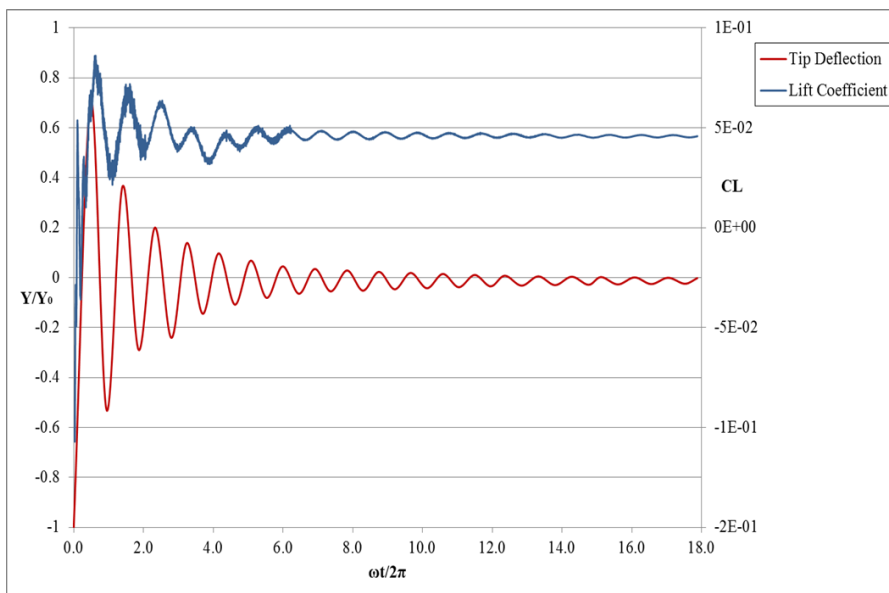


Figure 12 Comparison between Lift Coefficient and Tip Deflection vs. dimensionless time

Once obtained the time trend of the wing tip deflection, the damping factor ξ of the wing structure was determined. Applying the method of the logarithmic decrement δ , according to which

$$\delta = \ln \frac{A_n}{A_{n+1}} = 2\pi \frac{\xi}{\sqrt{1-\xi^2}} \quad (3.1)$$

where A_n and A_{n+1} are respectively the amplitudes of two successive peaks of the function, obtaining that the damping factor of the wing is $\xi = 0.103$.

The frequencies of the forcing and of the structural response are shown in Figure 13 and were obtained by calculating the Fast Fourier Transformation of the two functions. The comparison shows that the main frequency of the two signals is almost the same.

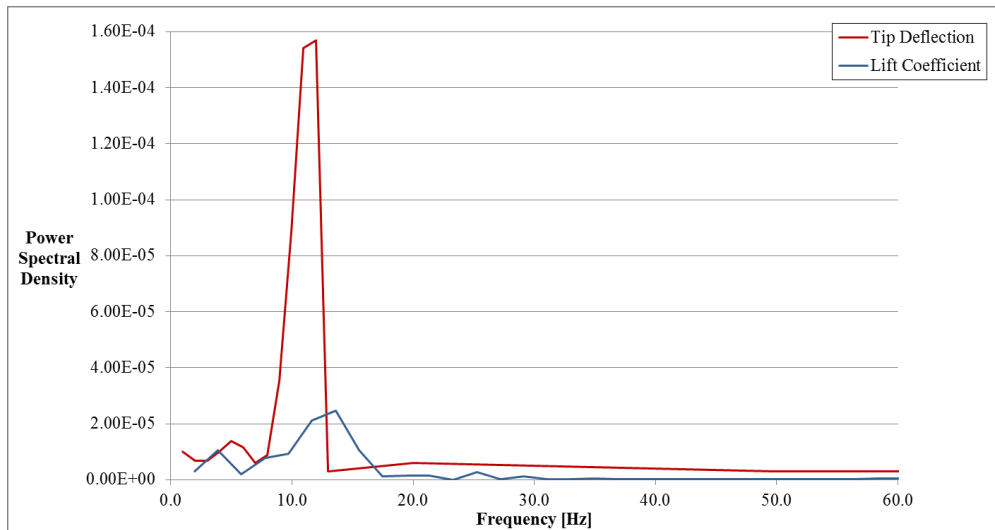


Figure 13 Comparison between PSD function of Lift Coefficient and Tip Deflection vs. Frequency

Finally in Figures 14 and 15 are shown the contours of pressure respectively on the surface of the wing and on a plane passing through the wing tip section at different instants of time.

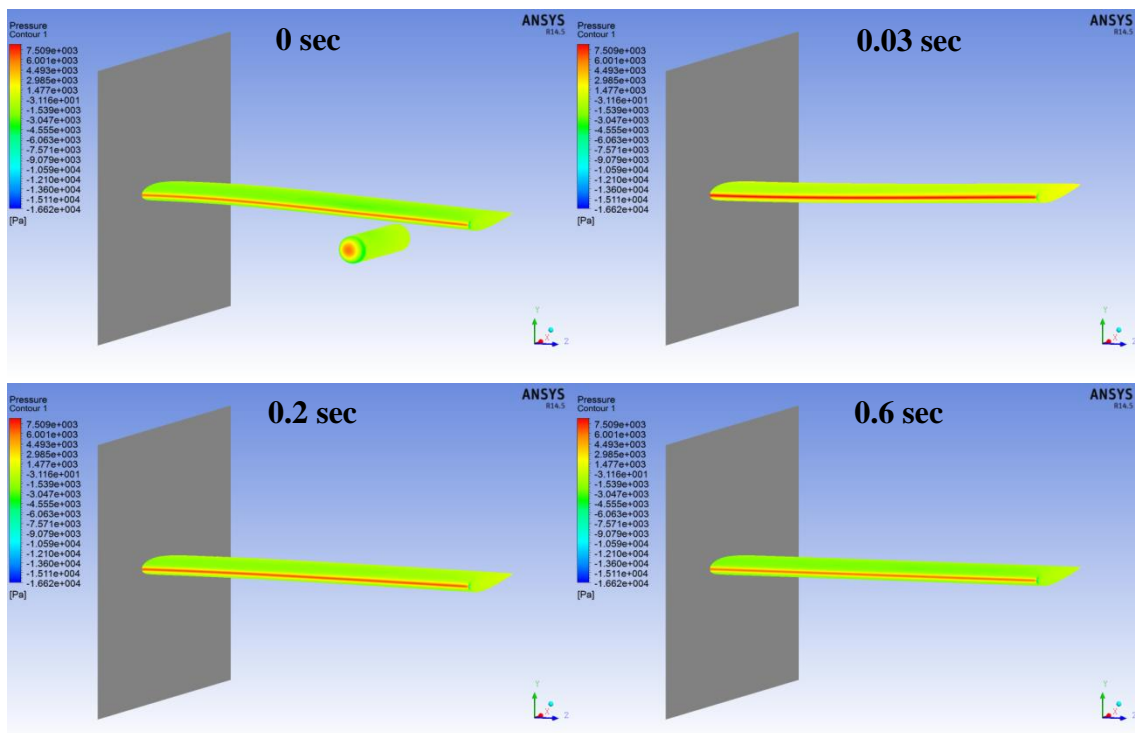


Figure 14 Pressure contours on the wing surface during the oscillation of the structure

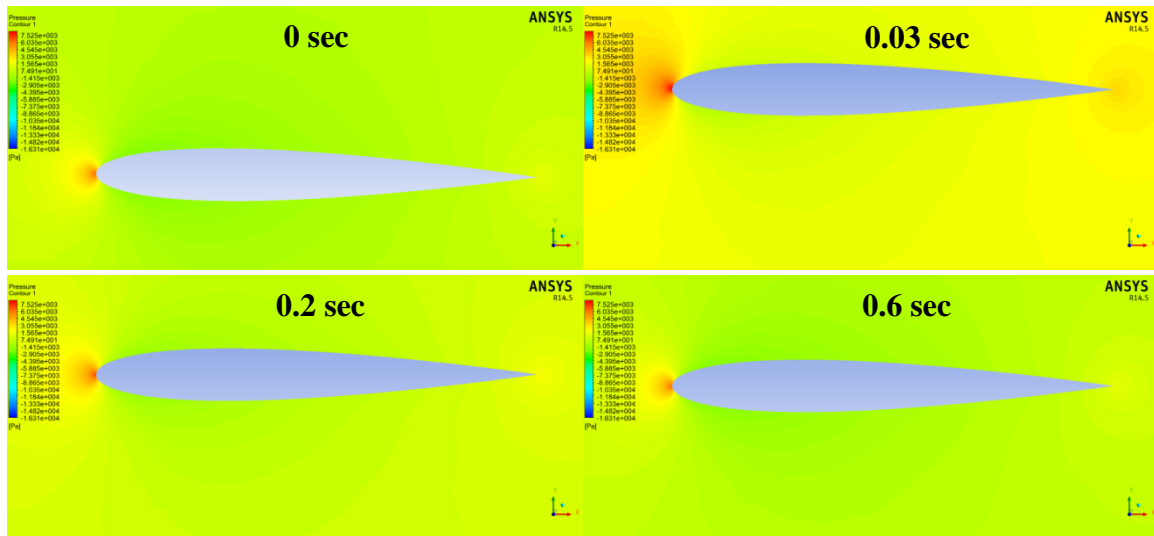


Figure 15 Pressure contours on wing tip section

4. Conclusions

The objective of this work was to develop a new methodology for the study of aeroelastic phenomena in the store separation problems, based on a modal superposition approach. The instruments used for this purpose were ANSYS Workbench, a tool for multi-physics analyses, and the mesh morpher RBF Morph. We started from the definition of a geometric model that has been used for the generation of a first mesh for fluid dynamic studies and a second one for the structural investigations; we performed a static structural analysis and a modal one; finally we accomplished CFD calculations for determining the deformation of the wing structure in elastic equilibrium. All this was done in a single computing environment, with the possibility of exchanging data between the various modules (ANSYS DesignModeler, Meshing, Fluent, Mechanical). The different deformations of the wing were obtained simply by implementing RBF Morph, in particular its FSI module, in Fluent solver.

The most important outcome is the reduction of the analysis time for a fluid-structure problem if compared with a full two-ways approach. Thanks to the modal FSI module of RBF Morph, all the work required at system coupling level for exchanging information between non-matching structural and fluid meshes is performed directly within the solver Fluent (i.e. performing the projection of pressure on modes and mesh deformation), greatly simplifying the process time. Moreover, we have demonstrated that the modal approach can be accommodated even for the complex load scenario related to the store separation problem.

The main goal for the future is surely to test this approach in cases characterized by finer meshes with detailed geometry typical of high fidelity CFD simulations.

5. Acknowledgements

Part of this work was financially supported by the RBF4AERO project funded by the EU's 7th Framework Programme (FP7-AAT, 2007 - 2013) under Grant Agreement no. 605396. We would like to express Our gratitude to Ubaldo Cella, Prof. Carlo de Nicola from the University of Naples "Federico II" and Cap. Pierluigi De Paolis, 1° divisione comando logistico - centro sperimentale di volo - Pratica di mare.

6. Bibliography

- M.E. Biancolini, 2011. *Fluid Structure Interaction with RBF Morph - A Generic Formula 1 Front End*. s.l., EnginSoft International Conference 2011.
- U. Cella, M.E. Biancolini, 2012. *Aeroelastic Analysis of Aircraft Wind-Tunnel Model Coupling Structural and Fluid Dynamic Codes*. Journal of Aircraft, 49(2), 407-414.
- J. R. Chambers, 2005. *Innovation in Flight*. NASA.
- L. Meirovitch, 2010. *Fundamentals of Vibrations*, Aveland Pr Inc, Reissue edition.
- M.E. Biancolini, E. Costa, C. Groth, F. Lagasco, 2013. *A mesh morphing based technique to efficiently perform FSI analyses for aeroelastic design applications*. AIDAA Conference 2013.
- S. Keye. *Fluid-structure coupled analysis of a transport aircraft and comparison to flight data*. 39th AIAA Fluid Dynamics Conference AIAA-2009-4198.
- A. de Boer, M.S. van der Schoot, H. Bijl, 2007. *Mesh deformation based on radial basis function interpolation*. Computers and Structures Vol. 85, Iss. 11-14, June - July, pp. 784-795.
- A.H. van Zuijlen, A. de Boer, H. Bijl, 2007. *Higher-order time integration through smooth mesh deformation for 3D fluid-structure interaction simulations*. Journal of Computational Physics Vol. 224, pp. 414-430.
- T.C.S. Rendall, C.B. Allen, 2010. *Reduced Surface Point Selection Options for Efficient Mesh Deformation Using Radial Basis Functions*. Journal of Computational Physics, 229, 2810-2820
- M.E. Biancolini, 2012. *Mesh Morphing and Smoothing by Means of Radial Basis Functions (RBF): A Practical Example Using Fluent and RBF Morph*, *Handbook of Research on Computational Science and Engineering: Theory and Practice*. Vol.2 pages 347-380.
- P. Chwalowski, J. Heeg, M. Dalenbring, A. Jirasek, M. Ritter, T. Hansen, 2013. *Collaborative HIRENASD analyses to eliminate variations in computational results*. International Forum on Aeroelasticity & Structural Dynamics, IFASD-2013-1D