AIAS – ASSOCIAZIONE ITALIANA PER L'ANALISI DELLE SOLLECITAZIONI
44° CONVEGNO NAZIONALE, 2-5 SETTEMBRE 2015, – UNIVERSITÀ DI MESSINA

AIAS 2015 - 503

STRUCTURAL OPTIMISATION USING ADVANCED RADIAL BASIS FUNCTIONS MESH MORPHING

R. Cenni[a,b], C. Groth[c], M.E. Biancolini[c]

[a] *SACMI Imola S.C. - Ceramic Engineering Department,*
Via Selice Provinciale 17/a, 40026 Imola, e-mail: riccardo.cenni@sacmi.it
[b] *Università degli studi di Modena e Reggio Emilia - Department of Engineering,*
Via Pietro Vivarelli, 10, 41125 Modena
[c] *Università degli Studi di Roma Tor Vergata – Dipartimento di Ingegneria dell'Impresa "Mario Lucertini", Via del Politecnico 1, 00133 Roma, e-mail: Corrado.groth@uniroma2.it; biancolini@ing.uniroma2.it;*

Sommario

L'ottimizzazione di forma è una tecnica comunemente utilizzata per migliorare le prestazioni strutturali dei componenti meccanici e strumenti basati su geometrie parametriche stanno diventando standard nel processo di ottimizzazione a livello industriale. Questi strumenti possono gestire diversi tipi di analisi strutturali ma sono limitati dalla necessità di garantire la coerenza geometrica. Aggiornare direttamente le posizioni nodali di modelli agli elementi finiti è una valida alternativa al re-meshing di geometrie parametriche, in grado di aiutare il progettista nella valutazione di alternative, studi ottimizzazione e progettazione robusta senza problemi di coerenza geometrica. Le Radial Basis Functions (RBF) sono riconosciute essere tra i migliori strumenti matematici per gestire il mesh morphing. Un' implementazione industriale di mesh morphing tramite RBF è il software **RBF Morph™** nato per applicazioni CFD e oggi disponibile anche in Ansys Mechanical come estensione sviluppata grazie al nuovo **Application Customization Toolkit** (ACT) di **Ansys®**. Presentiamo un'applicazione industriale di **SACMI** basata su **RBF Morph** e **Ansys Mechanical**: partendo da una topologia definita il lavoro mostra come il mesh morphing può essere usato per ottenere la forma migliore in termini di affidabilità del componente.

Abstract

It is well known that shape optimization is a way to improve the structural performance of components and tools based on parametric geometries are becoming standard for fine-tuning optimization processes in industry. These tools can handle different types of structural analysis, but they are limited by difficulties in maintaining geometry coherence. Direct update of nodal positions of finite element models by mesh morphing is a meaningful alternative to re-meshing a parametric geometry, helping the designer in what-if studies, optimization and robust design development without geometry coherence problems. Radial Basis Functions (RBF) are recognized among the best mathematical tools to perform mesh morphing. An industrial implementation of RBF is available in the software **RBF Morph™** born for CFD applications and now available in ANSYS Mechanical as an Extension created using the new **Application Customization Toolkit** (ACT) of **Ansys®**. We present an industrial application of **SACMI** based on **RBF Morph** and **Ansys Mechanical**: starting from a defined topology the paper shows how mesh morphing can be used to obtain the best shape in terms of component reliability.

Keywords: shape optimization, radial basis functions, mesh morphing, reliability.

1. INTRODUCTION

Currently, a lot of engineering enterprises have been concentrated to develop reliable, efficient and better products within least possible development times. To satisfy these multitudes of requirements, companies have now adopted structural optimization tools, like topology optimization and shape optimization in their design and development activities, due to obvious advantages. Topology optimization is used in the concept and predesign phase of the component design process. It reduces product development times by generating an optimal load compatible initial layout design. The innovative design proposal is achieved by removing material in form of holes from a maximum available delineated design space under the specified loads and constraints [1]. During the entire process location of nodes remain unchanged, whereas the material distribution is modified in each optimization iteration. Advanced topology optimization can be executed with different combinations of objective functions and constraints, multiple load cases and desired manufacturing restrictions. Once the topology is defined, the designer has to address the geometry of the component and shape optimization is an efficient way to do it. Today, geometry parameterization is almost a standard tool for industries and designers to evaluate alternative shapes in an automatic structured manner [2]. However, CAD-based optimizations are affected by some limitations: geometry coherence has to be preserved during optimization, re-meshing noise due to CAD reconstruction can affect FEA results and not all the geometrical features can be efficiently parameterized. It is well known that morphing techniques are used for shape optimization instead of CAD-based optimizations when parametric geometries are too complex to be properly managed [3]-[4]**.** Radial Basis Functions (RBF), at the core of the commercial Morphing Software RBF Morph, are well known [5] for the high local control and flexibility that comes with many advantages linked with their meshless nature. By means of the ANSYS ACT technology a brand new implementation of the acclaimed RBF Morph software is now available for Ansys Mechanical.

2. METHOD

2.1. Radial Basis Functions Interpolation

RBFs are powerful mathematical functions able to interpolate everywhere in the space data defined at discrete points only (source points). The interpolation quality and its behaviour between points depends on the kind of basis adopted. RBFs can be classified on the basis of the type of support (global or compact) they have, meaning the domain where the chosen RBF is non zero-valued [6]. Typical RBF functions are shown in Table 1. RBFs are scalar functions with the scalar variable r which, in the case of mesh morphing, can be assumed to be the Euclidean norm of the distance between two points defined in a three-dimensional space. In any case, a polynomial corrector is added to guarantee compatibility for rigid modes.

Radial Basis Functions with global support	$\varphi(r), r = \|\|r\|\|$
Spline type (R_n)	r^n, n odd
Thin plate spline (TPS_n)	$r^n \log(r)$, n even
Multiquadric (MQ)	$\sqrt{1+r^2}$
Inverse multiquadric (IMQ)	$\frac{1}{\sqrt{1+r^2}}$
Inverse quadratic (IQ)	$\frac{1}{1+r^2}$
Gaussian (GS)	e^{-r^2}

Radial Basis Functions with compact support	$\varphi(r) = f(\xi), \xi \leq 1, \xi = \frac{r}{R_{sup}}$
Wendland (C^0)	$(1-\xi)^2$
Wendland (C^2)	$(1-\xi)^4(4\xi+1)$
Wendland (C^4)	$(1-\xi)^6\left(\frac{35}{3}\xi^2+6\xi+1\right)$

Table 1: Typical RBF functions

A linear system (of order equal to the number of source point introduced [7]) needs to be solved for coefficients calculation. Operatively, once the RBF system coefficients have been calculated, the displacement of an arbitrary node of the mesh, either inside (interpolation) or outside (extrapolation) the domain, can be expressed as the sum of the radial contribution of each source point (if the point falls inside the influence domain). In such a way, a desired modification of the mesh nodes position (smoothing) can be rapidly applied preserving mesh topology. An interpolation function s composed by a radial basis φ and the aforementioned polynomial h of order $m-1$, where m is said to be the order of φ, is defined as follows if N is the total number of contributing source points.

$$s(x) = \sum_{i=1}^{N} \gamma_i \varphi\left(\left\|x - x_{k_i}\right\|\right) + h(x) \tag{1}$$

The degree of the polynomial has to be chosen depending on the kind of RBF adopted. A radial basis fit exists if the coefficients γ_i and the weight of the polynomial can be found such that the desired function values are obtained at source points and the polynomial terms give zero contributions at source points, that is:

$$s\left(x_{k_i}\right) = g_i, 1 \leq i \leq N \tag{2}$$

$$\sum_{i=1}^{N} \gamma_i p\left(x_{k_i}\right) = 0 \tag{3}$$

for all polynomials p with a degree less or equal than that of polynomial h. The minimal degree of polynomial h depends on the choice of the RBF. A unique interpolator exists if the basis function is a conditionally positive definite function [8]. If the basis functions are conditionally positive definite of order $m \leq 2$ [9] a linear polynomial can be used:

$$h(x) = \beta_1 + \beta_2 x + \beta_3 y + \beta_4 z \tag{4}$$

The subsequent exposition assumes that the aforementioned hypothesis is valid. A consequence of using a linear polynomial is that rigid body translations are exactly recovered. The values for the coefficients γ of RBF and the coefficients β of the linear polynomial can be obtained by solving the system:

$$\begin{pmatrix} M & P \\ P^T & 0 \end{pmatrix} \begin{pmatrix} \gamma \\ \beta \end{pmatrix} = \begin{pmatrix} g \\ 0 \end{pmatrix} \tag{5}$$

where g are the known values at the source points. M is the interpolation matrix defined calculating all the radial interactions between source points:

$$M_{ij} = \varphi\left(\left\|x_{k_i} - x_{k_j}\right\|\right), 1 \leq i \leq N, 1 \leq j \leq N \tag{6}$$

and P is a constraint matrix that arises balancing the polynomial contribution and contains a column of “1” and the x y z positions of source points in the others three columns:

$$P = \begin{pmatrix} 1 & x_{k_1} & y_{k_1} & z_{k_1} \\ 1 & x_{k_2} & y_{k_2} & z_{k_2} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{k_N} & y_{k_N} & z_{k_N} \end{pmatrix} \tag{7}$$

Radial basis interpolation works for scalar fields. For the smoothing problem, each component of the displacement field prescribed at the source points is interpolated as follows:

$$\begin{cases} s_x(x) = \sum_{i=1}^{N} \gamma_i^x \varphi(\|x - x_{k_i}\|) + \beta_1^x + \beta_2^x x + \beta_3^x y + \beta_4^x z \\ s_y(x) = \sum_{i=1}^{N} \gamma_i^y \varphi(\|x - x_{k_i}\|) + \beta_1^y + \beta_2^y x + \beta_3^y y + \beta_4^y z \\ s_z(x) = \sum_{i=1}^{N} \gamma_i^z \varphi(\|x - x_{k_i}\|) + \beta_1^z + \beta_2^z x + \beta_3^z y + \beta_4^z z \end{cases} \tag{8}$$

Radial basis method has several advantages that makes it very attractive in the area of mesh smoothing. The key point of is that, being a meshless method, only grid points are moved regardless of element connected and it is suitable for parallel implementation. In fact, once the solution is known and shared in the memory of each calculation node of the cluster, each partition has the ability to smooth its nodes without taking care of what happens outside because the smoother is a global point function and the continuity at interfaces is implicitly guaranteed. Furthermore, despite its meshless nature, the method is able to exactly prescribe known deformations onto the surface mesh: this effect is achieved by using all the mesh nodes as RBF centres with prescribed displacements, including the simple zero field to guarantee that a surface is left untouched by the morphing action.

2.2. Ansys Mechanical Extension

ANSYS$^{©}$ Workbench™ is built on a modular architecture that allows users to extend the functionality of such a framework using add-in software components. In particular, the ACT technology supplies internal mechanisms conceived to enable customizations of a Workbench application allowing for an easy connection with the inner libraries, thus allowing a deep integration with the system.
Exploiting this framework RBF Morph, well known for the morphing and shape optimization ground-breaking add-on system tailored for ANSYS FLUENT, is available also as an ACT extension for ANSYS Mechanical. Exploiting the features of ACT technology, the RBF Morph Extension is deeply integrated in Mechanical sharing its look & feel and interaction logic including the usual scoping tools for geometrical and mesh elements selections. The Extension can be enabled using the custom toolbar shown in Figure **1**: RBF Morph integration in the Mechanical tree, where also the access to the back to CAD features, morphing contours and additional information are located.

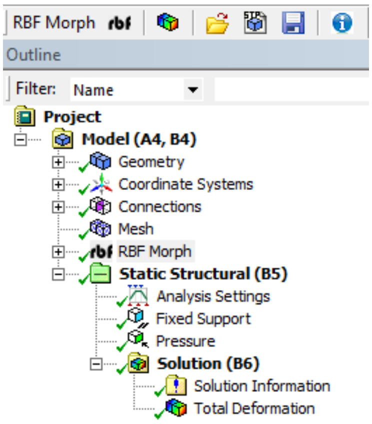


Figure 1: RBF Morph integration in the Mechanical tree

The extension is nested in the Mechanical tree as an added object right after the mesh element and before the solver, given its function of mesh modifier. To handle complex mesh modifications the

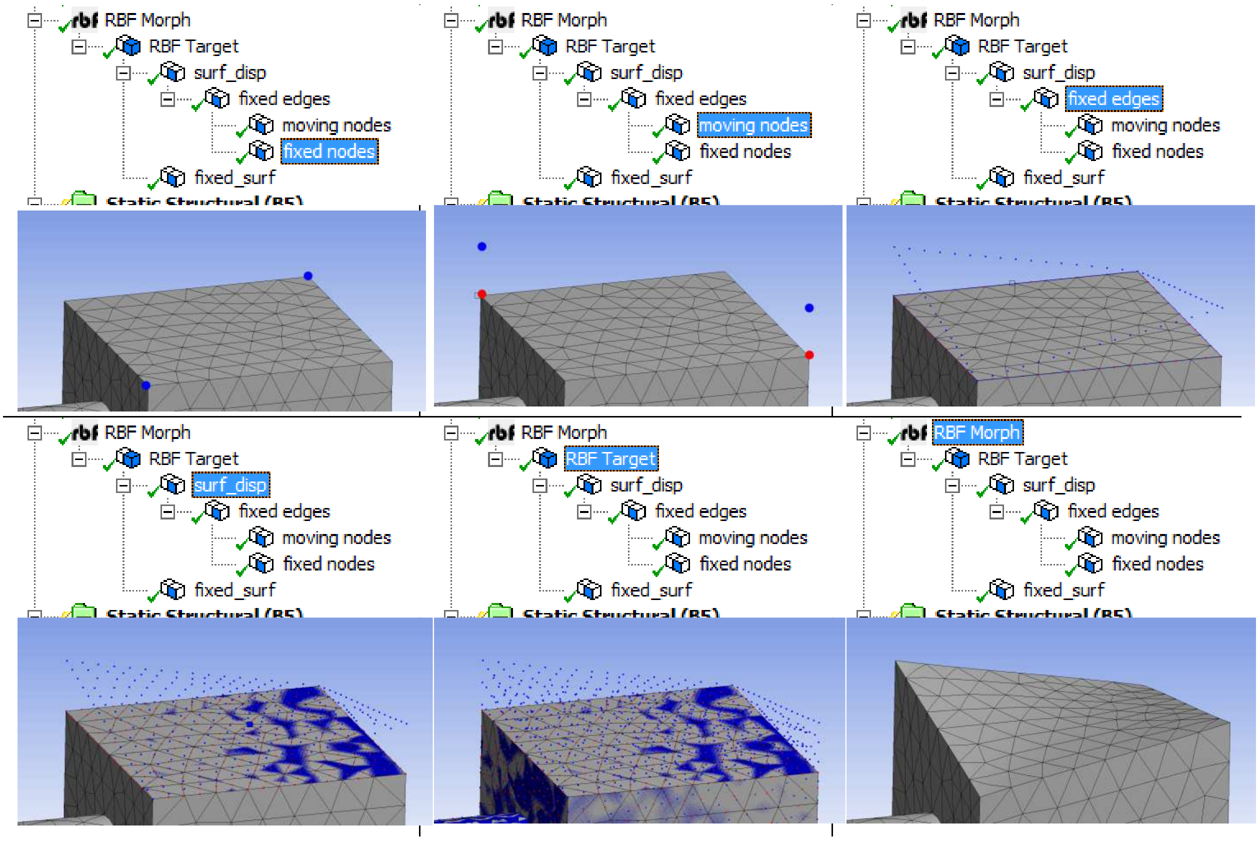
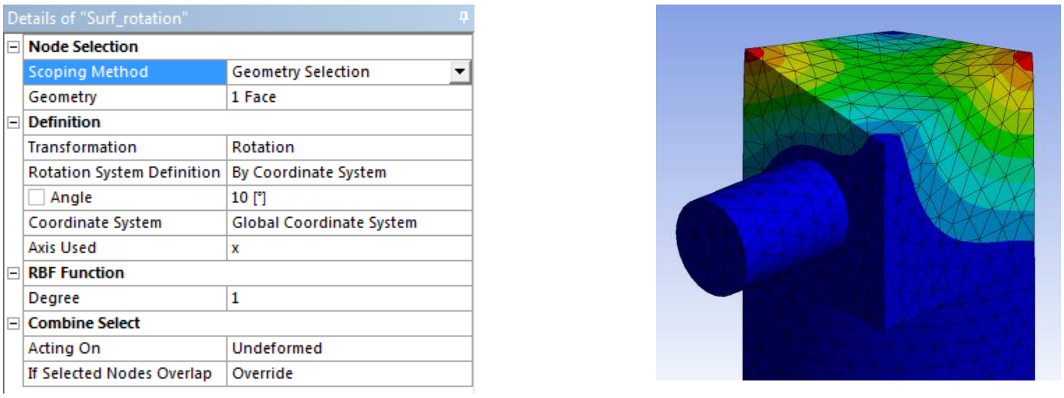


Figure 2: Preview and Morph for a hierarchical setup

RBF Morph Extension has a hierarchical working logic that foresees the use of multiple children as shown in Figure 2. Each children in the RBF Morph tree is a shape modifier acting on its selection (nodal, geometrical and named selections are available, this selection is called Source) and propagating it to its father (this selection is called Target). Multiple RBF Sources can be added to a target, as in the case of “moving nodes” and “fixed nodes” in Figure 2, where respectively moving nodes and fixed nodes are propagating their action to a set of fixed edges.



Details of "Surf_rotation"

Node Selection	
Scoping Method	Geometry Selection
Geometry	1 Face
Definition	
Transformation	Rotation
Rotation System Definition	By Coordinate System
☐ Angle	10 [°]
Coordinate System	Global Coordinate System
Axis Used	x
RBF Function	
Degree	1
Combine Select	
Acting On	Undeformed
If Selected Nodes Overlap	Override

Figure 3: left – rotation modifier, right – morphing contours

Several shape modifications are available including Translation, Rotation, Scaling, Curve and Surface Offset and Curve and Surface Targeting. For Translation, Scaling and Rotation custom coordinate systems can be selected for a faster setup as shown in Figure 3 left. Other available options are the kind of RBF used and the ability to apply the shape modification to the baseline mesh or to the already modified grid.

For each RBF source the resulting behaviour can be interactively previewed and, finally, the mesh morphed with a serial calculation or exploiting CUDA or OpenMP acceleration. The obtained morphed shape can be then analysed with a contour plot as shown in Figure 3 or exported back to cad format using the baseline NURBS geometry.

3. INDUSTRIAL TEST CASE

3.1. Problem overview

Crudely speaking, topology optimization is an useful computational method to distribute material in a given region, so that a given objective function is minimized while given constraints are respected. Even if commercial software are closer and closer to their customer' needs, a typical issue in using topology optimization seems to be the difficult in find the correct final proportion among component different zones. Moreover, in many cases, there is a desire to optimize a structure, taking stresses or fatigue analysis outputs into account, but issues emerge when trying to introduce local derived quantities in topology optimization. It is well known that stress constraints can lead to so called "singular topologies" [10]-[11] and that the local nature of stress constraints implies the introduction of mathematical norms and constraints aggregation. In order to rightly consider stresses and fatigue analysis, a forward step is needed in structural optimization: the shape optimization. Typical workflows of structural optimization provide local shape optimization after topology to get local optimum results: thanks to fine tuning optimization the analyst is able to work on geometry details in particular areas. Geometries of massive components obtained with topology optimization are an example of complex geometries hardly manageable with CAD parameters and we retain that morphing technique is currently the best choice to manage the shape obtained by a topology optimization.

We present here an industrial example of using RBF Morph ACT to increase structural performances of a SACMI component. The studied component is a massive cast-iron part which reliability is evaluated via an in-house fatigue analysis criterion completely developed inside Ansys Workbench environment. Choosing RBF Morph technology gives the Ansys Mechanical users' the capability of working inside their standard design tools by giving a new advanced tool to their hands.

We obtained the starting geometry to perform shape optimization by topology optimization from Altair Optistruct. The basics of this preliminary step are described in section “Topology definition” while section “Morphing setup and results” describes the details in using RBF Morph to perform shape optimization on different zones together with optimization results.

3.2. Topology definition

We perform the following topology optimization to get the starting geometry for shape optimization:

$$\begin{cases} min\ C(\boldsymbol{x}) \\ st \begin{cases} u_i(\boldsymbol{x}) < u_{lim,i},\ i = 1, \dots, n_{uc} \\ \sum_{e=1}^{n_e} m_e(\boldsymbol{x}) \le m_{lim} \\ SF_g(\boldsymbol{x}) > SF_{lim} \\ \epsilon < x_e < 1,\ e = 1, \dots, n_e \end{cases} \end{cases} \tag{9}$$

where C is the component compliance, $\boldsymbol{x}$ the array of topology design variables, x_e, m_e is the mass of element e, n_e the number of elements in design space, m_{lim} the mass limit, u_i the displacement of node i, $u_{lim,i}$ the displacement limit for node i, n_{uc} the number of node displacement constraints, $SF_g(x)$ the global Safety Factor measure and SF_{lim} the global Safety Factor limit. The global Safety Factor was evaluated by using the multi-axial Dang-Van criterion for fatigue analysis implemented in Altair Optistruct evaluated as a global constraint on the design space. No local stress nor fatigue constraints are allowed in the design space but global averaged constraints are available and useful [12].

Figure **4** shows in purple the design space and in grey the no-design space of topology optimization domain at first *(a)* and last *(b)* optimization iterations. We re-design the CAD-model obtained from topology optimization *(c)* by adding some functional features and used this geometry as starting point for RBF Morph shape optimization.

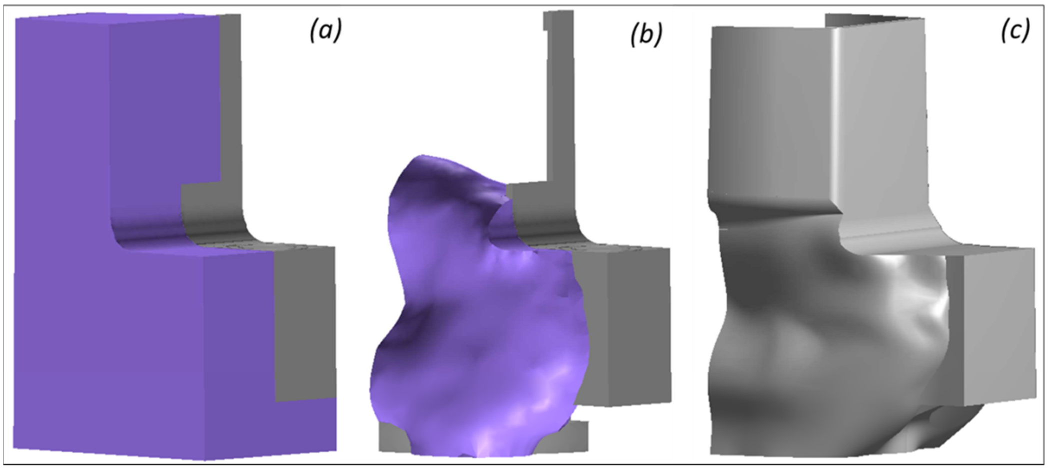


Figure 4: Design and No-Design space of topology optimization at first and last iteration and re-designed CAD-model

3.3. Morphing setup and results

Starting from the same geometry obtained via topology optimization, we investigated different zones with RBF Morph for Ansys Mechanical and morphing setup and results are provided for each investigated zone. Even if they are not provided here, combinations of the proposed morphing setup could also be implemented in order to better explore the full design space.

3.3.1. Internal clew

The zone affected by morphing is circled in Figure 5*(a)* and the RBF Morph setup is shown in Figure 5*(b-d)*. The aim of morphing this zone is to evaluate the best shape of the main internal clew considering different zones fatigue results. Besides the internal clew, also the fillet has been changed in order to get a more smoothed external surface. In Figure 5 and in the analogous for the other examples, the parametric morphing sources are highlighted in yellow and the fixed sources in red. We decided to use the translation feature acting on the central edge of the upper face of the clew to parameterize the morphing setup by fixing the edges on the other face side. In general it is possible to define a generic translation direction by using coordinate system definition and different setup could be possible to achieve a similar morphing result: e.g. edge offset, edge scaling, surface scaling. We used the scaling feature to the external fillet in order to replace it with a smooth face.

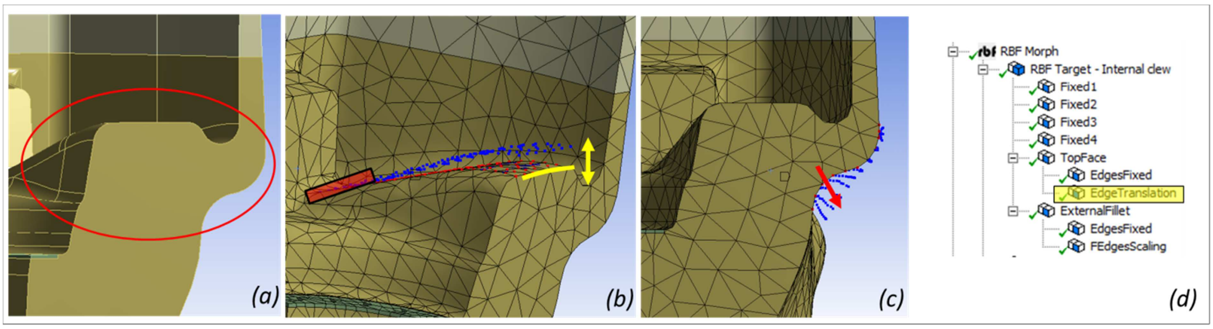


Figure 5: Internal clew original geometry *(a)*, RBF Morph previews *(b)* and *(c)* and RBF Morph tree *(d)*. Parametric features in yellow.

In the current example we performed the following multi-objective shape optimization:

$$\begin{cases} max\, SF_1 \\ min\, \sigma_2 \\ min\, \sigma_3 \\ min\, u_4 \\ st\ P1_{l,lim} < P1 < P1_{u,lim} \end{cases} \tag{10}$$

where SF_1 is the minimum Factor of Safety of zone 1 evaluated with SACMI in-house multi-axial fatigue criterion, σ_2 and σ_3 the maximum of the fatigue equivalent stresses of zones 2 and 3 respectively, u_4 the deformation of a relevant point of zone 4. In this example no nodes of the four zones of the component where optimization objectives are evaluated are directly moved by morphing. We used the MOGAII genetic algorithm implemented in modeFrontier [13] to find the Pareto frontier of this problem. Figure 6 shows optimization results of the current example: bubble 4D chart and parallel coordinate chart for each Pareto frontier point. All the solutions belonging to the Pareto frontier are optimal form the mathematical point of view, however, considering the different importance of the different objectives, the analyst should choose the best compromise between possible optimal solutions. For instance, if SF_1 and u_4 are more important than σ_2 and σ_3 a well-balanced compromise could be the yellow design point of Figure **6**, or vice versa a light blue design should be chosen.

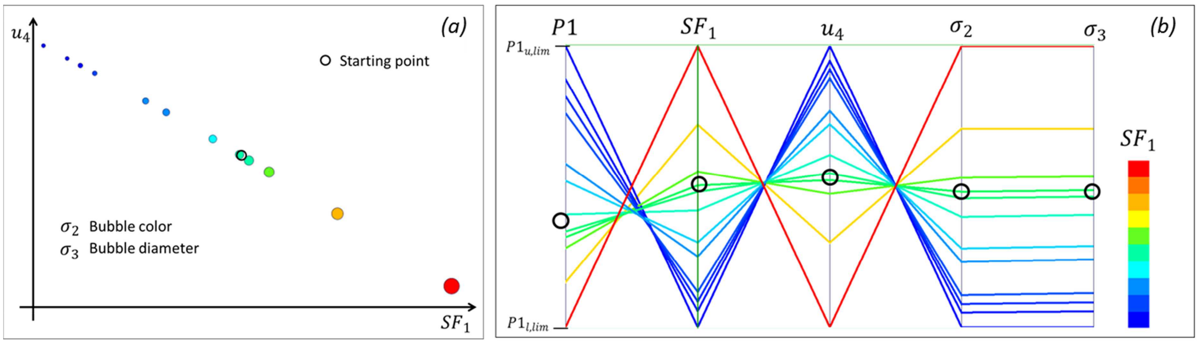


Figure 6: Optimization results: bubble 4D chart *(a)* and parallel coordinate chart *(b).*

3.3.2. External fillet

Fillets are typical zones of stress concentrations and low reliability, they are usually easy to manage with parametric geometries especially if only one radial dimension is used to define the fillet. Considering multi-radius fillet could be possible in some cases but usually difficulties in maintaining geometry coherence arise. In the example we provide here, the fillet feature is governed by moving a node path as RBF source while keeping fixed the fillet border edges. This simple setup gives the possibility to modify the fillet face curvature without any problem in geometry coherence. More complicated setup could also be possible in order to obtain different fillet shape: e.g. combination of scaling and translation of source nodes, combination of face offset and source nodes translation.

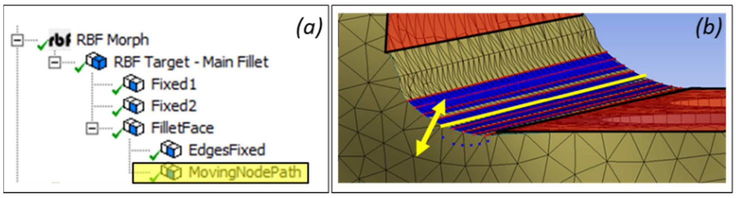


Figure 7: RBF Morph tree *(a)* and graphical morphing description *(b)*. Parametric features in yellow.

We perform the following mono-objective shape optimization in order to enhance reliability of the external fillet zone:

$$\begin{cases} \max(minSF) \\ st \begin{cases} P1_{l,lim} < P1 < P1_{u,lim} \\ P2_{l,lim} < P2 < P2_{u,lim} \end{cases} \end{cases} \tag{11}$$

where SF is the Factor of Safety of the external fillet and $P1$ and $P2$ are the parameters governing the node path scaling. It is a max(min) mono-objective optimization: we choose the SIMPLEX gradient based optimization algorithm [14] to maximize fillet Factor of Safety by limiting the possible changes within the morphing parameters limits depicted in Figure **8**.

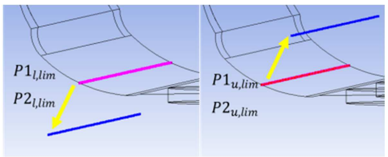


Figure 8: RBF Morph preview of MovingNodePath source with $\boldsymbol{P1}$ and $\boldsymbol{P2}$ limits.

This is a typical case of using morphing to act directly on the output control zone. Figure 9 shows optimization results of the current example: the objective value point and the parallel coordinate chart for each optimization design point. Thanks to described shape optimization, we get an increase of 16% in terms of Factor of Safety in the studied zone.

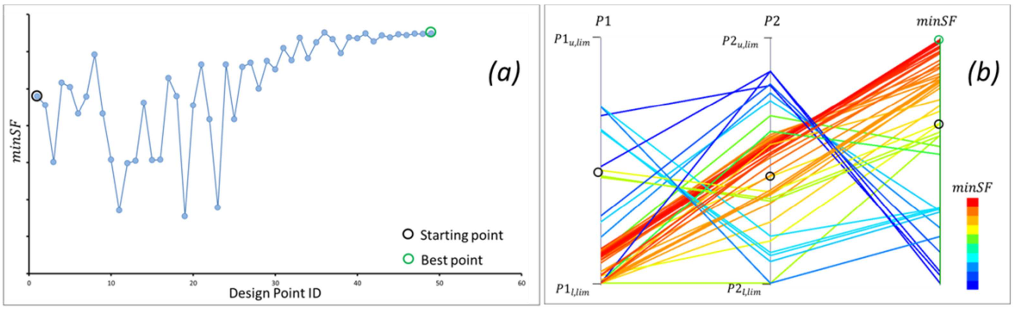


Figure 9: Optimization results: Objective history chart *(a)* and parallel coordinate chart *(b).*

3.3.3. External clews

As-cast surfaces could be very irregular, however their shapes should be commiserate to foundry requirements and dimensional measuring systems capabilities. An important role of mesh morphing is the one of redefining complex shapes by smoothing them in order to maintain certain reliability requirement together with easier manifacturing. We provide an example here in using mesh morphing as a sort of automatic push & pull tool to manage the bottom zone and the external front face of the studied component. Figure 10 displays the morphing setup.

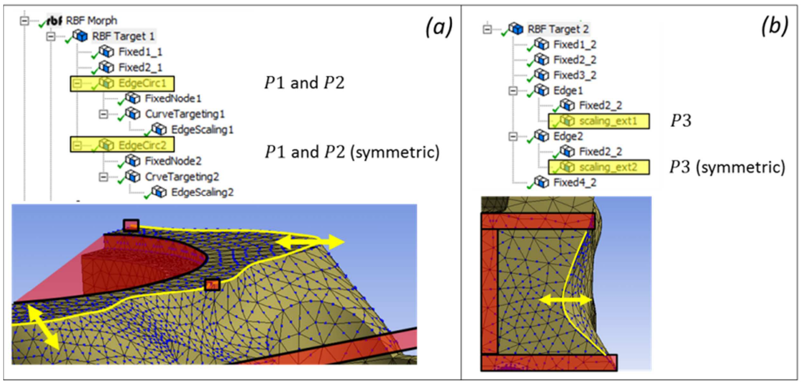


Figure 10: RBF Morph tree and graphical morphing description for the bottom zone *(a)* and the external front face *(b)*. Parametric features in yellow.

In the current example we performed the following multi-objective shape optimization:

$$\begin{cases} max\, SF_1 \\ min\, \sigma_2 \\ min\, u_3 \\ st \begin{cases} P1_{l,lim} < P1 < P1_{u,lim} \\ P2_{l,lim} < P2 < P2_{u,lim} \\ P3_{l,lim} < P3 < P3_{u,lim} \end{cases} \end{cases} \tag{12}$$

where SF_1 is the minimum Factor of Safety of zone 1 evaluated with SACMI in-house multi-axial fatigue criterion, σ_2 is the maximum of fatigue equivalent stresses of zone 2 and u_3 the deformation of a relevant point of zone 3. $P1$ and $P2$ are the parameters governing the edge scaling of the bottom zone while $P3$ rules the external front face scaling. We considered the representative FEA results of three different relevant zones to automatic push & pull the described zones via the MOGAII genetic optimization algorithm. The result of the optimization are displayed in Figure **11** where bubble chart and parallel coordinate graph are provided for each design point: we used an optimization strategy based both on direct FEA calculations and Response Surface (RSM) evaluations to better cover the full design space. Since it is a multi-objective optimization, the analyst should choose the component final shape among the Pareto frontier points by ranking the objectives.

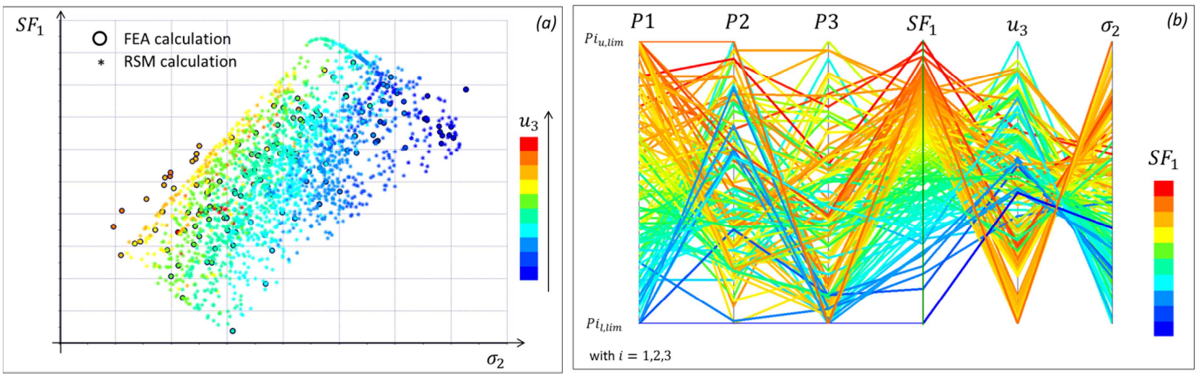


Figure 11: Optimization results: bubble chart *(a)* and parallel coordinate chart *(b).*

4. CONCLUSIONS

Advanced radial basis functions morphing of RBF Morph has been used to re-shape the result obtained from a topology optimization with Altair Optistruct. We used different features to manage the RBF source points and we performed shape optimization driven both by SACMI in-house fatigue criterion and typical FEA results achieving interesting results. We believe that the combination of advanced optimization tools like topology and shape is a valid opportunity for industries looking for innovative design processes. The integration of RBF Morph in Ansys Mechanical gives to the Workbench user the possibility to manage complex finite element models and perform shape optimization studies directly inside a familiar environment.

REFERENCES

[1] M. P. Bendsoe, O. Sigmund, *Topology Optimization, Theory Methods and applications*, Springer Verlag, 1-47 (2003).

[2] A. Falk, F.J. Barthold, E. Stein. “A hierarchical design concept for shape optimization based on the interaction of CAGD and FEM”, *Structural Optimization*, 18, 12-23 (1999).

[3] Y. Ding, “Shape optimization of structures: a literature survey”, *Computers & Structures*, 24:6, 985-1004, (1986).

[4] A. Schleupen, K. Maute, E. Ramm. “Adaptive FE-procedures in shape optimization”, Structural and Multidisciplinary Optimization, 19, 282-302, (2000).

[5] M.E. Biancolini, “Mesh Morphing and Smoothing by Means of Radial Basis Functions (RBF): A Practical Example Using Fluent and RBF Morph”, *Handbook of Research on Computational Science and Engineering: Theory and Practice*, IGI Global, ISBN13: 9781613501160, (2011)

[6] A. De Boer, M.S. Van der Schoot, H. Bijl, Mesh deformation based on radial basis function interpolation, Computers and Structures, Vol. 85, Nos. 11-14, 2007, pp. 784-795

[7] M.D. Buhmann, *Radial Basis Functions*, Cambridge University Press, New York (2003)

[8] C. Micchelli, *Interpolation of scattered data: Distance matrices and conditionally positive definite functions*, Constructive Approximation, Vol. 2, No. 1, pp. 11-22 (1986).

[9] A. Beckert, H. Wendland, *Multivariate interpolation for fluid-structure-interaction problems using radial basis functions*, Aerospace Science and Technology, Vol. 5, No. 2, pp. 125-134 (2011)

[10] U. Kirsch, “On singular topologies in optimum structural design”. *Structural optimization*, 2, 1333-142 (1990).

[11] G.I.N. Rozvany, “On design-dependent constraints and singular topologies”, *Structural and Multidisciplinary Optimization*, 21(2), 164-172 (2001).

[12] Altair Optistruct user’s manual. Release 13.0

[13] S. Poles, “MOGA-II An improved Multi-Objective Genetic Algorithm”, *ESTECO Technical Report*, 006 (2003).

[14] S. Poles, “The SIMPLEX Method”, *ESTECO Technical Report*, 005 (2003).