Multiphysics numerical investigation of the aeroelastic stability of a Le Mans Prototype car

Corrado Groth[a] , Andrea Chiappa[a] , Stefano Porziani[a] , Marco E. Biancolini[a,*] , Emanuele Jacoboni[b] , Elisa Serioli[b] , Franco Mastroddi[c]

[a] University of Rome "Tor Vergata", Department of Enterprise Engineering "Mario Lucertini", Via del Politecnico 1, Rome 00133, Italy

[b] Dallara, Via Provinciale 33, Varano de' Melegari 43040, Italy

[c] University of Rome "La Sapienza", Department of Mechanical and Aerospace Engineering, Via Eudossiana 18, Rome 00184, Italy

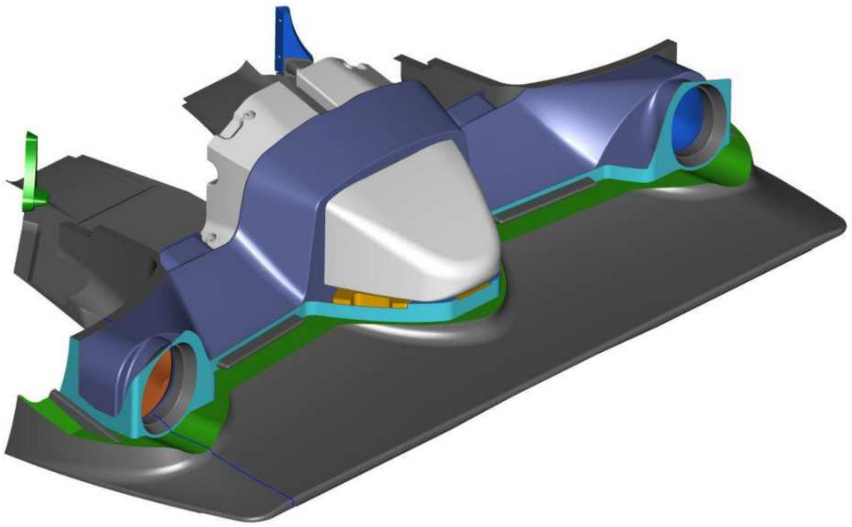
AIAS2019 - 4-7 Settembre 2019, ASSISI (PG)

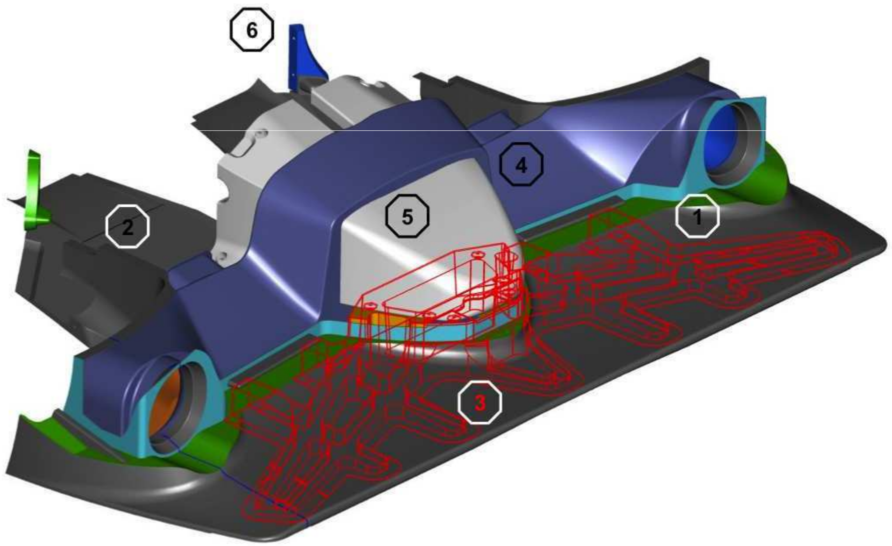
Introduction

- Flutter analysis carried out on the front wing splitter mounted on the 2001 Le Mans Prototype car by Dallara
- During test drives the driver experienced, at a given velocity, an irregular behaviour of the front assembly:
  Front wing flutter instability?
- Stiffening spider added empirically: IT WORKS! Why?
- Modal frequencies are increased and flutter velocity is shifted outside the vehicle speed range

Introduction

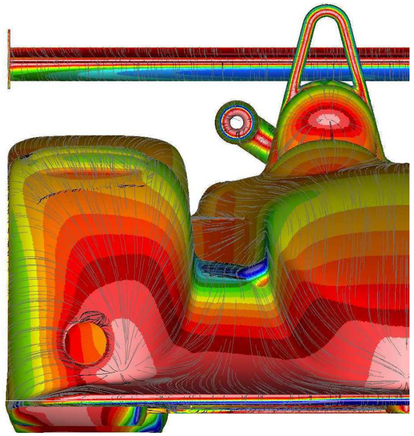
- Flutter analysis carried out on the front wing splitter mounted on the 2001 Le Mans Prototype car by Dallara

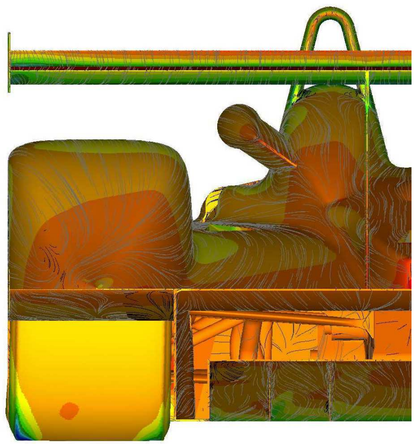




Summary

- Theoretical background
  - Modelling the aeroelastic problem
  - Time input for building the aerodynamic ROM
  - Radial Basis Functions mesh morphing
  - Flutter analysis
- Application: Dallara LMP1 front wing splitter
- Results
- Conclusions





Modelling the aeroelastic problem

- A generic displacement field can be described as a linear superposition of its modal shapes

$$X(x,t) = \sum_{i=0}^{n} N_i(x)\, q_i(t)$$

- The system equation of motion can be then simplified exploiting mode orthogonality

$$[M]\,\{\ddot{q}(t)\} + [C]\,\{\dot{q}(t)\} + [K]\,\{q(t)\} = \{Q(t)\}$$

- {Q(t)} is the Generalized Aerodynamic Force vector (GAF)

Modelling the aeroelastic problem

- Moving to the Laplace domain, the equation of motion can be rewritten

$$[M]\{\ddot{q}(t)\} + [C]\{\dot{q}(t)\} + [K]\{q(t)\} = \{Q(t)\} \longrightarrow \left([M]\,s^2 + [C]\,s + [K]\right)\{q(s)\} = \{Q(s)\}$$

- Under the hypothesis of linearized aerodynamics, the GAF vector can be written in function of the modal coordinates

$$\{Q(s)\} = q_\infty\,[H(p)]\,\{q(s)\} \qquad p := sL_a/V_\infty$$

- Where [H(p)] is the GAF transfer function matrix. The i-th column of the GAF frequency response matrix can be estimated as

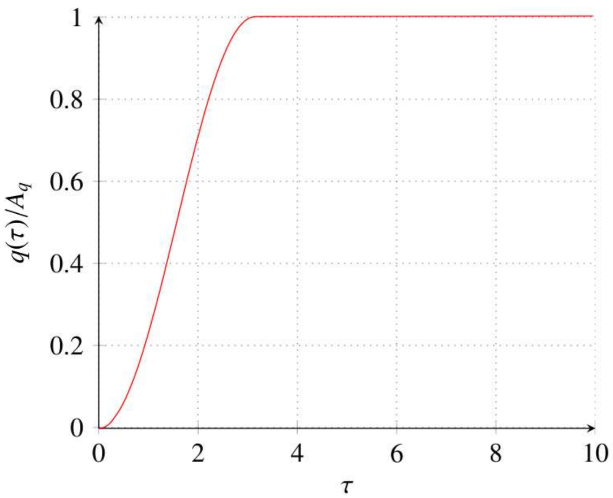
$$[H(\omega;V_\infty)|_i] = \frac{\mathcal{F}\left(\{Q(t)\}^{(i)}\right)}{\mathcal{F}\left(q(t)_i\right)} \qquad k := \omega L_a/V_\infty$$

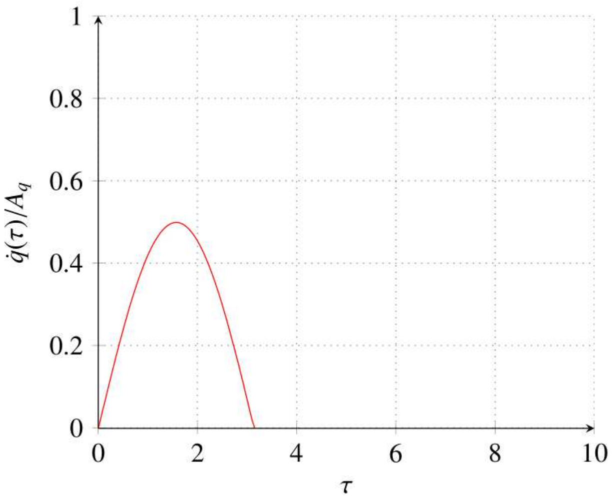
Time input for building the aerodynamic ROM

- A single FSI static simulation is needed to reach the trimmed configuration, then mode shapes are excited in turn to evaluate the GAF response matrix
- Choice of proper time law of motion not a trivial task:
  - Numerical noise?
  - Small displacements?
- Typical choices:
  - Harmonic law
  - Impulsive law
  - Step function law

Time input for building the aerodynamic ROM

- In this work a smoothed step function is used
- Problems linked with discontinuities are removed by using a smoothed function. A coarser time discretization can be also employed to correctly catch the fluid transient response





$$\tau = tV_\infty / L_a$$

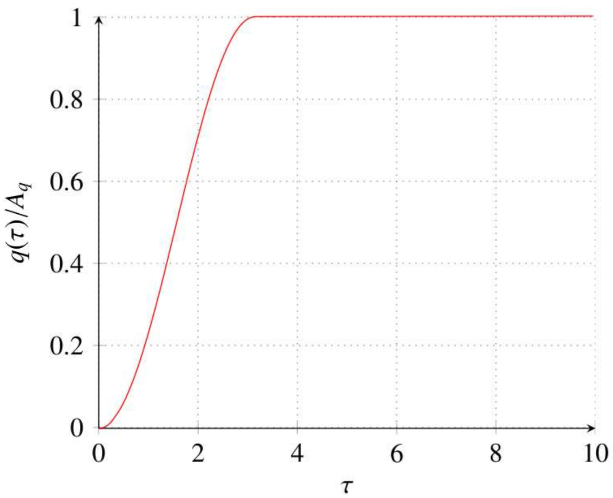
Time input for building the aerodynamic ROM

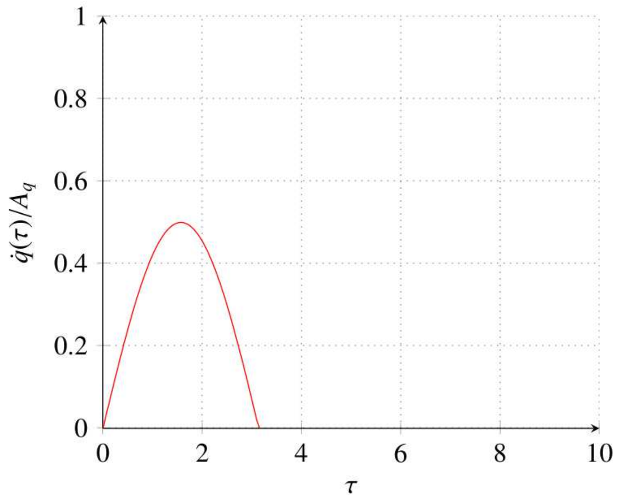
$$A_q = \frac{4\epsilon L_a}{d_{max} k_{max}}$$

$$k = [0, k_{max}]$$

$$q_i(\tau) = \begin{cases} \frac{A_q}{2} \cdot \left[1 - \cos\left(k_q \tau\right)\right] & \text{if } \tau < \tau_q \\ A_q & \text{if } \tau \geq \tau_q \end{cases}$$

$$\dot{q}_i(\tau) = \begin{cases} \frac{A_q k_q V_\infty}{2L_a} \cdot \left[1 - \sin\left(k_q \tau\right)\right] & \text{if } \tau < \tau_q \\ 0 & \text{if } \tau \geq \tau_q \end{cases}$$



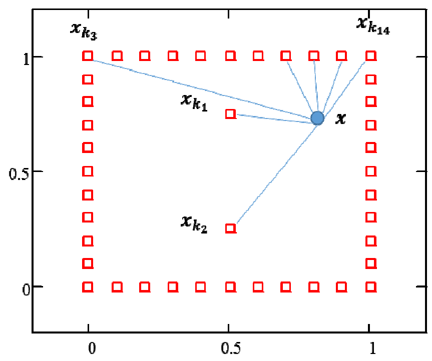


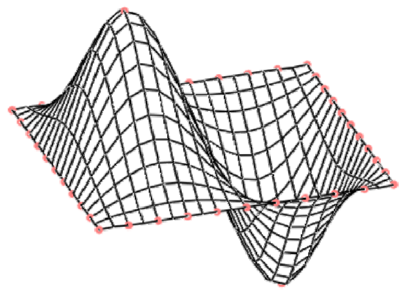
$$\tau = tV_\infty / L_a$$

Radial Basis Functions mesh morphing

- RBFs are a mathematical tool capable to interpolate at a generic point in the space a function known at a discrete set of points (source points)
- Particularly suitable to deform a grid, interpolating mesh deformation

$$s(\boldsymbol{x}) = \sum_{i=1}^{N} \overbrace{\gamma_i \varphi\left(\underbrace{\left\| \boldsymbol{x} - \boldsymbol{x}_{k_i} \right\|}_{\text{distance from the i-th source point}}\right)}^{\text{radial basis}} + \overbrace{h(\boldsymbol{x})}^{\text{polynomial}}$$





Radial Basis Functions mesh morphing

- If evaluated on the source points, the interpolating function gives exactly the input values:

$$\begin{matrix} s(\boldsymbol{x}_{\boldsymbol{k}_i}) = g_i \\ h(\boldsymbol{x}_{\boldsymbol{k}_i}) = 0 \end{matrix} \quad 1 \le i \le N$$

- The RBF problem is associated to the solution of the linear system:

$$\begin{bmatrix} \mathbf{M} & \mathbf{P} \\ \mathbf{P}^{\mathrm{T}} & 0 \end{bmatrix} \begin{pmatrix} \boldsymbol{\gamma} \\ \boldsymbol{\beta} \end{pmatrix} = \begin{pmatrix} \boldsymbol{g} \\ 0 \end{pmatrix} \qquad M_{ij} = \varphi\left(\boldsymbol{x}_{k_i} - \boldsymbol{x}_{k_j} \right) \qquad 1 \le i, j \le N \qquad \mathbf{P} = \begin{bmatrix} 1 & x_{k_1} & y_{k_1} & z_{k_1} \\ 1 & x_{k_2} & y_{k_2} & z_{k_2} \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} \\ 1 & x_{k_N} & y_{k_N} & z_{k_N} \end{bmatrix}$$

Radial Basis Functions mesh morphing

- Once solved the RBF problem, each displacement component is interpolated
- Quality and behaviour of the interpolation depends on the basis function employed. Compactly and globally supported functions are available

$$
\begin{cases}
s_x(\boldsymbol{x}) = \sum_{i=1}^{N} \gamma_i^x \varphi\left(\boldsymbol{x} - \boldsymbol{x}_{k_i}\right) + \beta_1^x + \beta_2^x x + \beta_3^x y + \beta_4^x z \\
s_y(\boldsymbol{x}) = \sum_{i=1}^{N} \gamma_i^y \varphi\left(\boldsymbol{x} - \boldsymbol{x}_{k_i}\right) + \beta_1^y + \beta_2^y x + \beta_3^y y + \beta_4^y z \\
s_z(\boldsymbol{x}) = \sum_{i=1}^{N} \gamma_i^z \varphi\left(\boldsymbol{x} - \boldsymbol{x}_{k_i}\right) + \beta_1^z + \beta_2^z x + \beta_3^z y + \beta_4^z z
\end{cases}
$$

RBF	φ(r)	RBF	φ(r)
Spline type (Rn)	r^n, n odd	Inverse multiquadratic (IMQ)	$\frac{1}{\sqrt{1+r^2}}$
Thin plate spline	$r^n \log(r)$ n even	Inverse quadratic (IQ)	$\frac{1}{1+r^2}$
Multiquadratic (MQ)	$\sqrt{1+r^2}$	Gaussian (GS)	e^{-r^2}

Flutter analysis

- Flutter is:
  - From a physical point of view a dynamic instability and can be seen as a positive feedback between body deflections and fluid dynamic loads
  - From a mathematical point of view the instability study of its linearized and time-invariant system around an equilibrium condition
- Splitting the load term into trimmed-related and unsteady loads, flutter analysis consists in the determination of the poles S and the associated eigen-vectors {w} (n + 1 unknowns) satisfying the following n algebraic equations

$$\left([M]\, s^S + [C]\, s + [K] - \frac{1}{2}\varrho_\infty V_\infty^2 \,[H(s;V_\infty)]\right)\{w\} = 0$$

Flutter analysis

- That can be rewritten as:

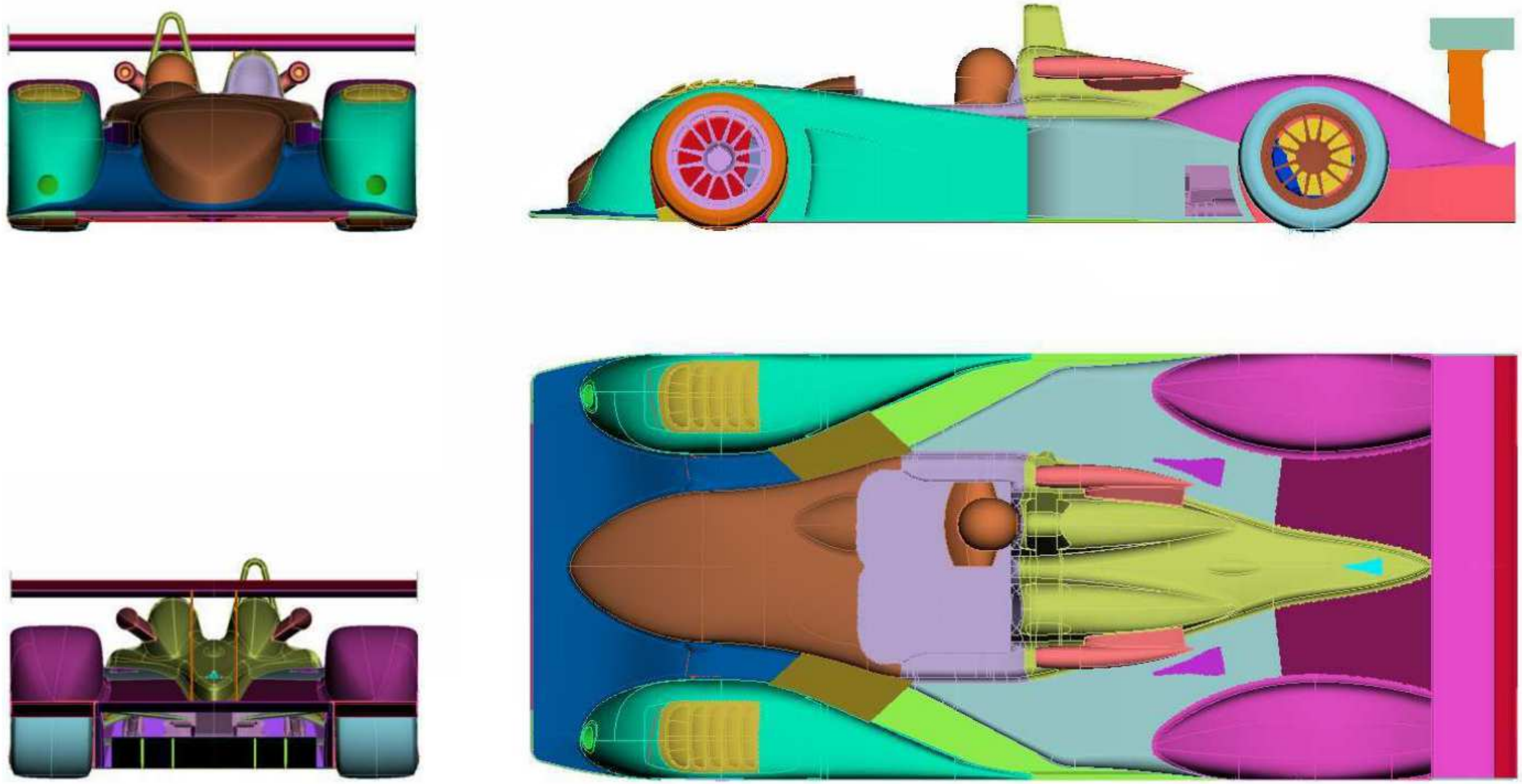
$$\left([M]\, s^S + [C]\, s + [K] - \frac{1}{2}\varrho_\infty V_\infty^2 \left[H(s; V_\infty)\right]\right)\{w\} = 0 \quad \longrightarrow \quad [F(s, V_\infty)]\,\{w\} = 0$$

- n equations in n+1 unknown must be solved, so a closing criteria such as a normalization must be employed

$$[F(s, V_\infty)]\,\{w\} = 0$$
$$\{w\}^T\,[W]\,\{w\} = c$$

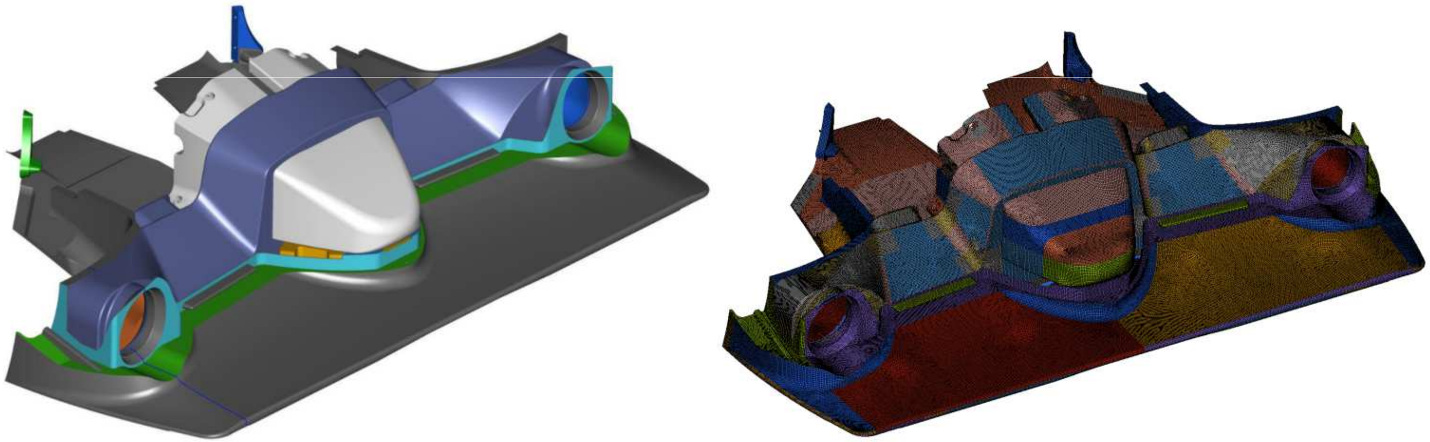
- Where [W] is a diagonal weight matrix and c is an arbitrary constant. The Newton-Raphson method was employed

Application



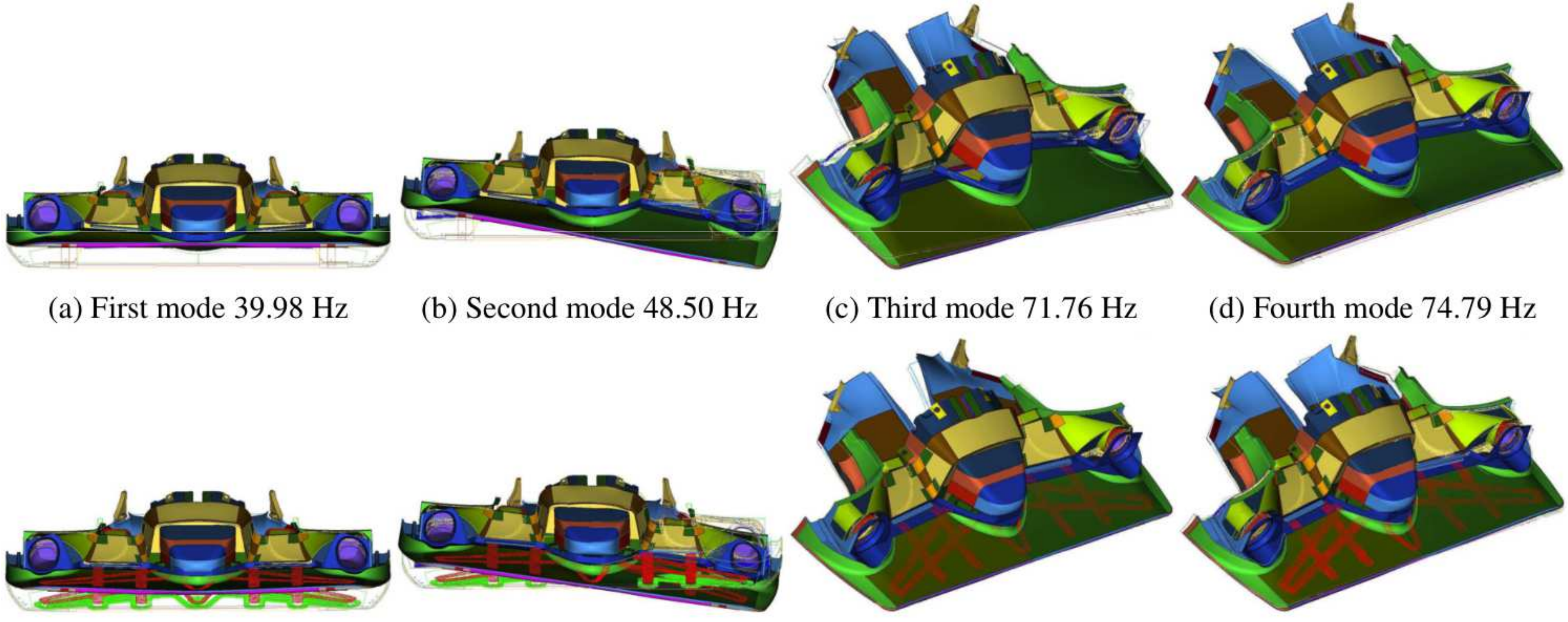
Application

- Flow around the vehicle was investigated on the whole geometry, but only the front wing portion was considered as wetted surface for the FSI study



Structural eigenvalue problem on 400k shell elements FEM model

- Four modes were extracted for the two configurations studied



(a) First mode 39.98 Hz (b) Second mode 48.50 Hz (c) Third mode 71.76 Hz (d) Fourth mode 74.79 Hz

(e) First mode 49.40 Hz (f) Second mode 53.89 Hz (g) Third mode 79.50 Hz (h) Fourth mode 81.77 Hz

Application

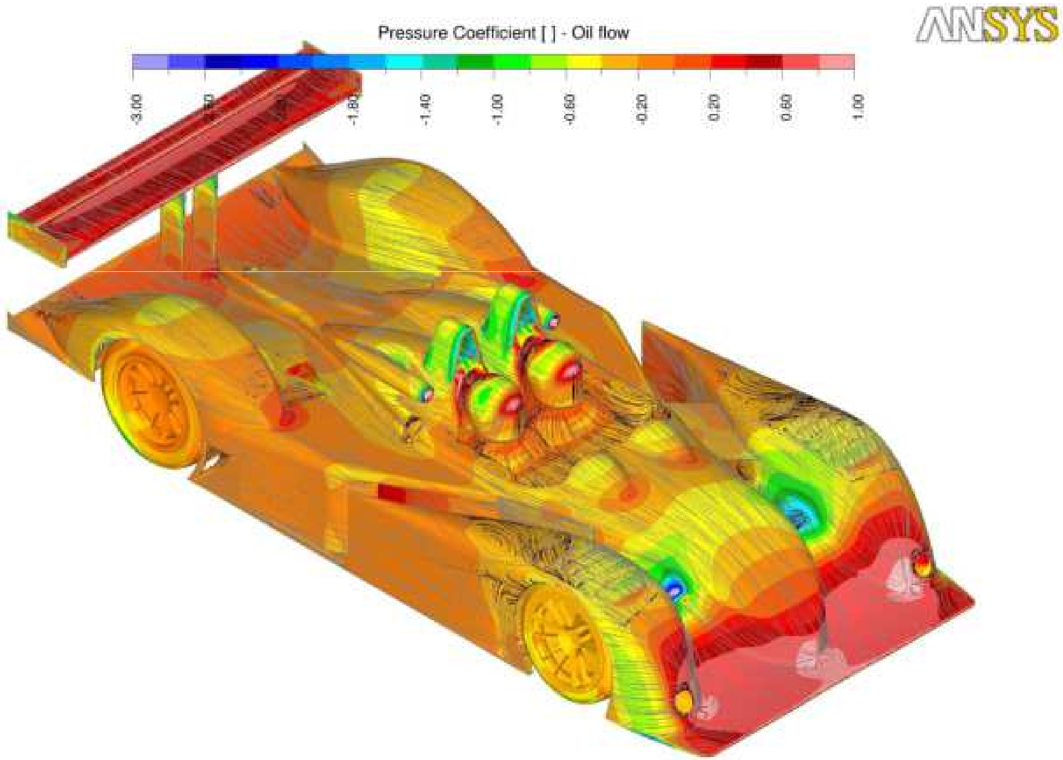
- Stiffened model with higher eigenvalues as expected.
- Modal Assurance Criterion (MAC) employed to study eigenvector differences between the two models

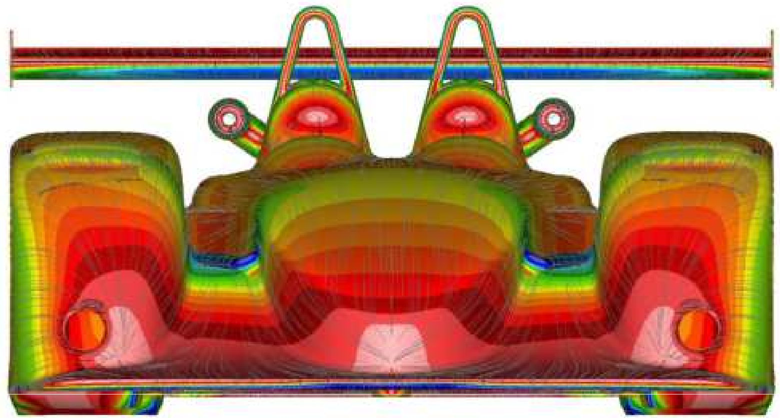
0.915	0.379	0.082	0.300
0.397	0.982	0.102	0.343
0.494	0.503	0.135	0.445
0.574	0.540	0.158	0.492

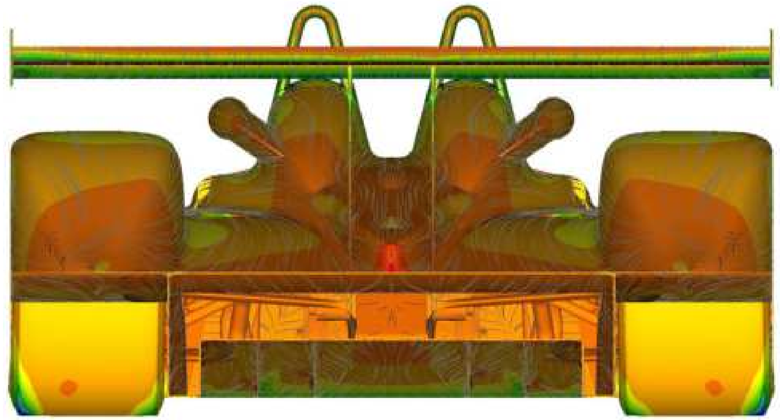
- CFD analysis conducted using k – epsilon and air incompressible
- Different velocities were taken into account, from 40 m/s to 100 m/s.
- To accelerate the CFD evaluation a half-vehicle symmetric domain comprised of about 240M cells was employed

Application

- 50 m/s results for pressure coefficients

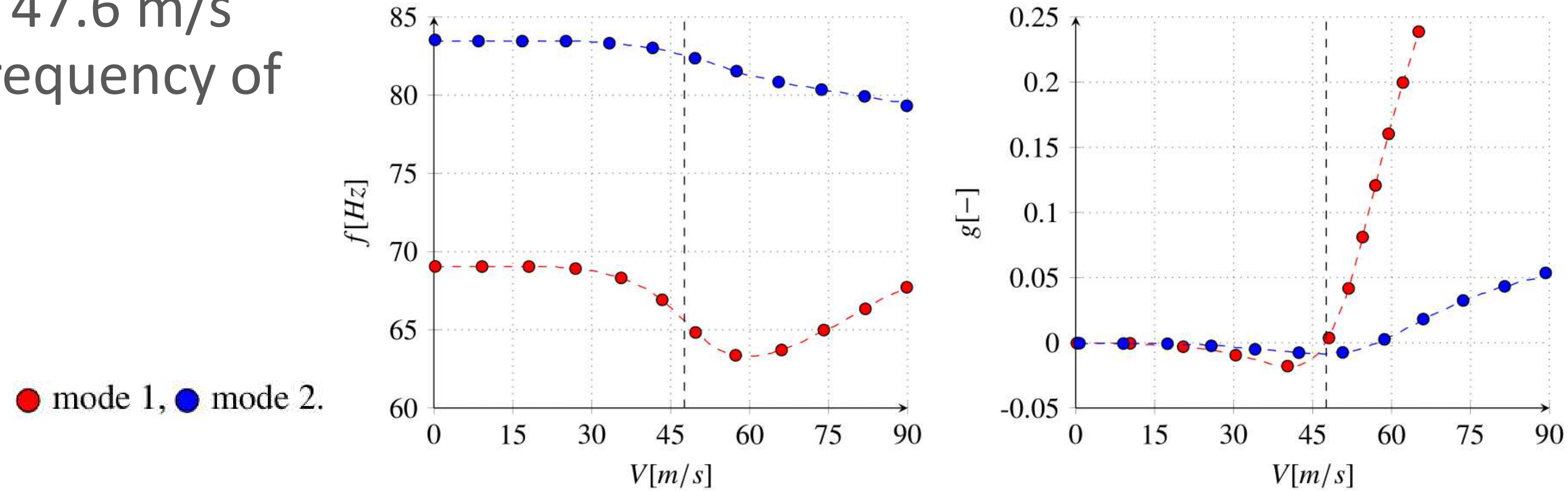






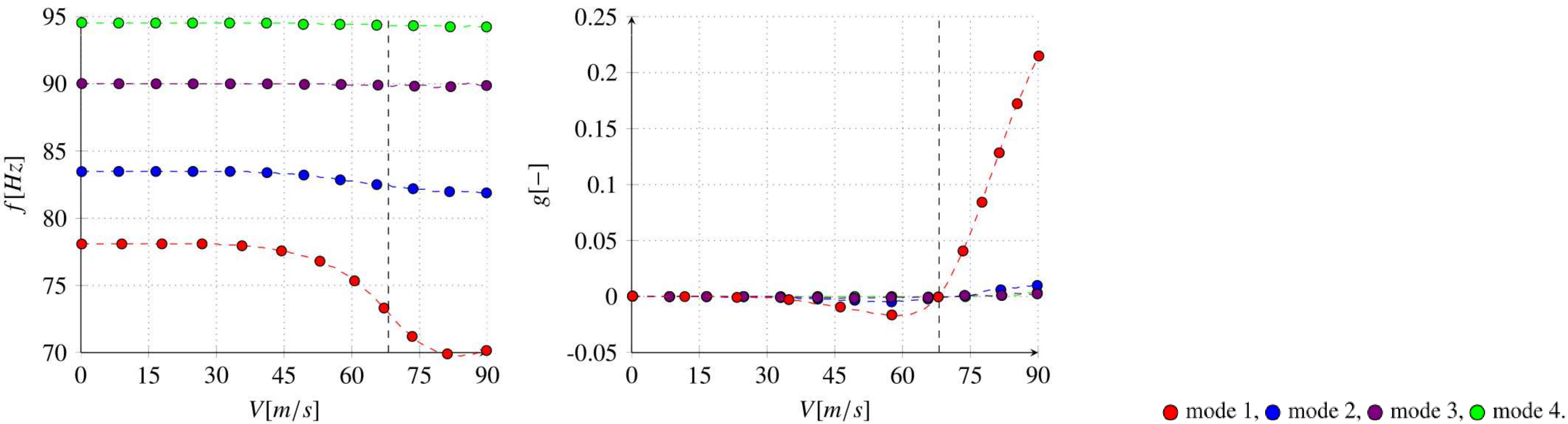
Results

- For flutter analysis the Newton-Raphson method employed using cubic splines to interpolate the Aerodynamic matrix known at discrete velocities and reduced frequencies
- Plotting baseline V-f and V-g diagrams (where g the norm of the real and imaginary parts of the complex pole s) a flutter instability can be seen for 47.6 m/s with a frequency of 65.7 Hz



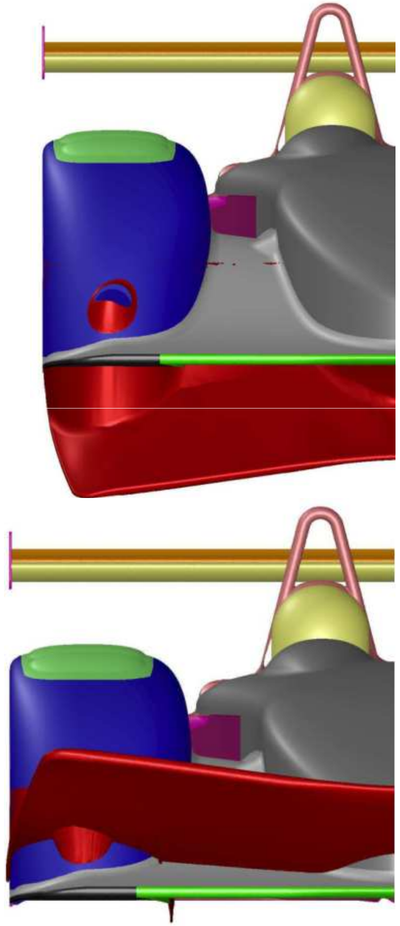
Results

- With the stiffened configuration, using four modes, instabilities occur at 68.1 m/s and at a frequency of 73.1 Hz.
- As expected a flutter frequency increase is catched



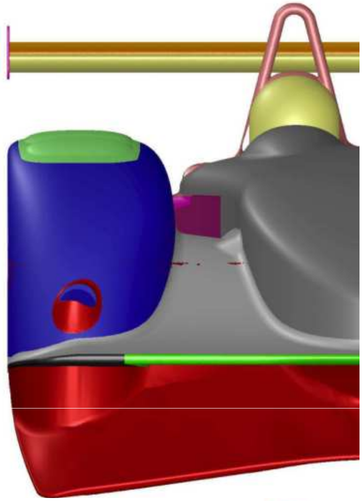
Conclusions

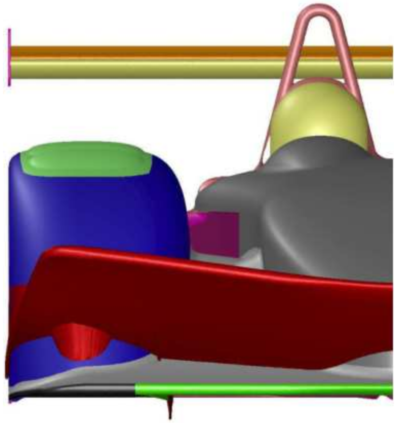
- The flutter analysis of the front wing splitter mounted on the 2001 Le Mans Prototype car by Dallara (LMP1) was presented
- A modal superposition coupling was employed using RBF mesh morphing.
- Vibration modes were excited employing a smoothed step function.
- The flutter problem was solved for the baseline and modified configuration using the aerodynamic transfer function matrices.



Conclusions

- The critical speed experimentally observed to be in the operating range of the car was satisfactorily captured by the model, underestimating of 13.9 m/s baseline and stiffened configurations.
- Differences could be ascribed to the structural model boundary conditions and to the removal of the wheels in the CFD to accelerate calculation
- Refined CFD and FEM models are needed as a future development to achieve a better comparison with experimental results





Multiphysics numerical investigation of the aeroelastic stability of a Le Mans Prototype car

THANK YOU FOR THE ATTENTION

Corrado Groth, Andrea Chiappa[a], Stefano Porziani[a], Marco E. Biancolini[a,*], Emanuele Jacoboni[b], Elisa Serioli[b], Franco Mastroddi[c]