Optimisation of industrial parts by mesh morphing enabled automatic shape sculpting

Stefano Porziani[a], Corrado Groth[a], Luca Mancini[a], Riccardo Cenni[b], Matteo Cova[b], Marco E. Biancolini[a]

[a]University of Rome “Tor Vergata”, Rome 00133, Italy

[b]Sacmi Imola S.C., Via Prov.le Selice 17/a, Imola 40026, Ital

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Introduction

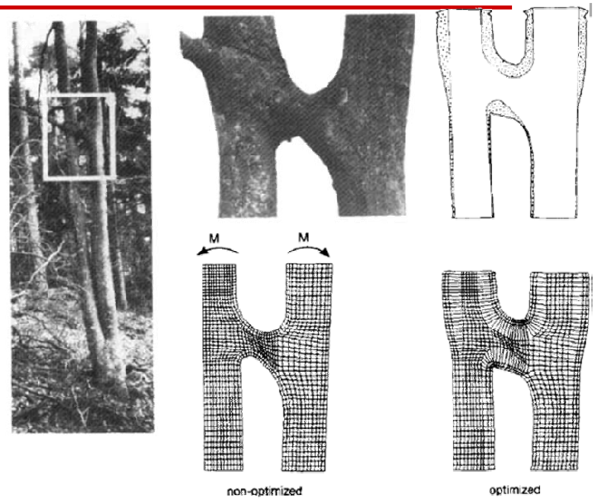
- Mechanical component **optimization** is a paramount target in every engineering application.
- A valuable tool for optimization in complex load and constraint configuration is the **Finite Element Method** (**FEM**), which allows to test **different configurations** before the prototyping phase.
- Both procedures, parameter based and BGM, require the **generation** of **additional FEM models**: this task can be very **time-consuming** specially dealing with complex shape components.
- To overcome this problem **Mesh morphing** can be adopted: it allows to generate new FEM models without modifying the geometry and without the need to remesh it.

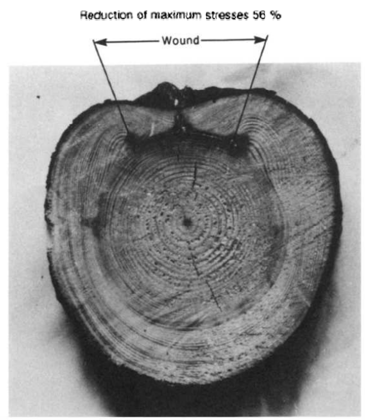
Introduction

- Mesh morphing can be adopted to set-up automatic shape optimization using it in conjunction with **Biological Growth Method** (**BGM**) and **Adjoint Method**. Thanks to this synergy a **high grade of automation** can be achieved.
- Both approaches exploit data coming from numerical analysis to define a shape modification that will sculpt model surfaces so that stress levels are optimized.
- In the present work, the tool adopted for morphing the **FEM** mesh is **RBF Morph™**, which is based on Radial Basis Functions (**RBFs**). FEM pre-processor and solver used is ANSYS® Mechanical™.

BGM Background

- **BGM** approach is based on the observation that **biological** structures growth is driven by **local** level of **stress**.
- Bones and trees' trunks are able to **adapt the shape** to mitigate the stress level due to external loads.
- The process is driven by stress **value at surfaces**. Material can be **added or removed** according to local values.
- Was proposed by Mattheck & Burkhardt in 1990*





*Mattheck C., Burkhardt S., 1990. A new method of structural shape optimization based on biological growth. Int. J. Fatigue 12(3):185-190.

BGM Background

- The BGM idea is that surface growth can be expressed as a **linear law** with respect to a given threshold value:

$$\dot{\varepsilon} = k\left(\sigma_{Mises} - \sigma_{ref}\right)$$

- Waldman and Heller* refined this first approach proposing a **multi peak** one:

$$d_i^j = \left(\frac{\sigma_i^j - \sigma_i^{th}}{\sigma_i^{th}}\right) \cdot s \cdot c\,, \qquad \sigma_i^{th} = max(\sigma_i^j) \text{ if } \sigma_i^j > 0 \quad \text{or} \quad \sigma_i^{th} = min(\sigma_i^j) \text{ if } \sigma_i^j < 0$$

- In **RBF Morph ANSYS Workbench ACT extension** a different implementation is present and different **stress types** can be used to modify the surface shape:

$$S_{node} = \frac{\sigma_{node} - \sigma_{th}}{\sigma_{max} - \sigma_{min}} \cdot d$$

Stress/strain type	Equation	Stress/strain type	Equation
von Mises stress	$\sigma_e = \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$	Stress intensity	$\sigma_e = max(\lvert\sigma_1 - \sigma_2\rvert, \lvert\sigma_2 - \sigma_3\rvert, \lvert\sigma_3 - \sigma_1\rvert)$
Maximum principal stress	$\sigma_e = max(\sigma_1, \sigma_2, \sigma_3)$	Maximum Shear stress	$\sigma_e = 0.5 \cdot (max(\sigma_1, \sigma_2, \sigma_3) - min(\sigma_1, \sigma_2, \sigma_3))$
Minimum principal stress	$\sigma_e = min(\sigma_1, \sigma_2, \sigma_3)$	Eqv. plastic strain	$\varepsilon_e = [2(1+\nu')]^{-1} \cdot \left(0.5\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2}\right)$

*Waldman W., Heller M., 2015. Shape optimization of holes in loaded plates by minimization of multiple stress peaks, Defence Science and Technology Organisation Fisherman Bend, Australia, Aerospace Div, http://www.dtic.mil/docs/citations/ADA618562.

Adjoint Background

- **Adjoint** method allows to obtain the **sensitivities** of an **objective function** with respect to a set of **input parameters**.
- This can be applied to the three displacement for each node of the computational mesh, so that a **shape modification** can be obtained.
- Adjoint is successfully used in Computational Fluid-Dynamics (CFD) but can be also applied in Computational Structural Mechanics (CSM).
- It is possible to differentiate the discritised equation (**Discrete Adjoint method**) or to derive equation prior to their differentiation (**Continuous Adjoint method**).

Adjoint Background

- Considering the discrete case, the objective function can be expressed as function of displacement and the derived:

$$\Psi = f(\mathbf{X}(u), u) \qquad \frac{d\Psi}{du} = \frac{\partial \Psi}{\partial u} + \frac{\partial \Psi}{\partial \mathbf{X}}\frac{\partial \mathbf{X}}{\partial u}$$

- To obtain the displacement $\frac{\partial \mathbf{X}}{\partial u}$ two methods are available: the **direct** one (which has to be re-evaluated for each input parameter) and the **adjoint** one (which need only one calculation no matter how many input parameters are used) which uses a Lagrange-like multiplier to obtain displacements:

$$\mathbf{K}\mathbf{X} = \mathbf{F} \;\Rightarrow\; \mathbf{K}\lambda = \frac{\partial \Psi}{\partial \mathbf{X}}^{\mathrm{T}} \;\Rightarrow\; \frac{d\Psi}{du} = \frac{\partial \Psi}{\partial u} + \lambda^{\mathrm{T}}\left(\frac{\partial \mathbf{F}}{\partial u} - \mathbf{X}\frac{\partial \mathbf{K}}{\partial u}\right)$$

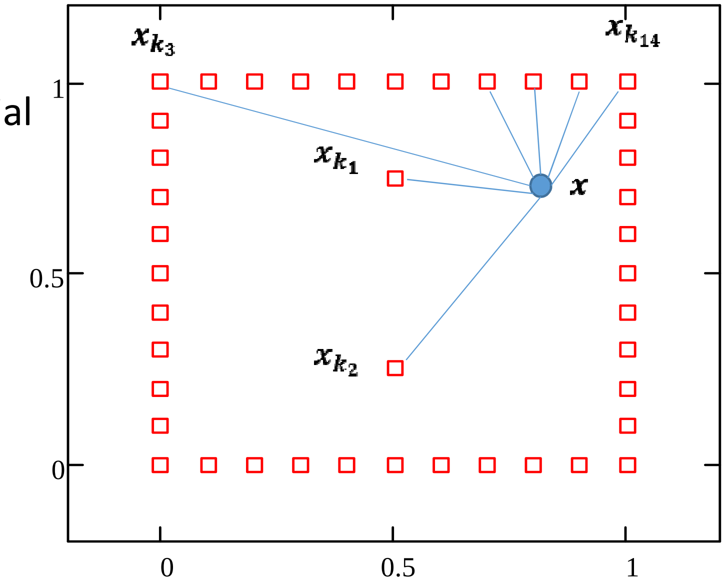
RBF Background

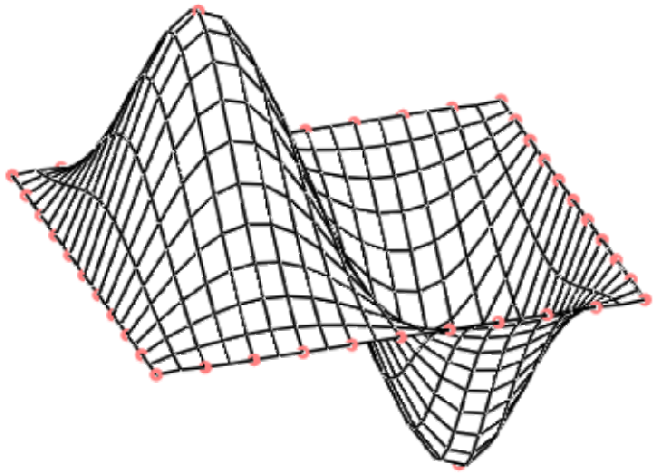
- RBFs are a mathematical tool capable to **interpolate** in a generic point in the space a function **known** in a discrete set of points (**source points**).
- The interpolating function is composed by a **radial basis** and by a **polynomial**:

radial basis — polynomial

$$s(\boldsymbol{x}) = \sum_{i=1}^{N} \gamma_i \varphi\left(\left\| \boldsymbol{x} - \boldsymbol{x}_{k_i} \right\|\right) + h(\boldsymbol{x})$$

distance from the i-th source point





RBF Background

- If evaluated on the source points, the interpolating function gives exactly the input values:

$$s(\boldsymbol{x}_{k_i}) = g_i \qquad 1 \le i \le N$$
$$h(\boldsymbol{x}_{k_i}) = 0$$

- The RBF problem (evaluation of coefficients γ and β) is associated to the solution of the linear system, in which **M** is the interpolation matrix, **P** is a constraint matrix, **g** is the vector of known values on the source points:

$$\begin{bmatrix} \mathbf{M} & \mathbf{P} \\ \mathbf{P}^{\mathrm{T}} & 0 \end{bmatrix} \begin{pmatrix} \boldsymbol{\gamma} \\ \boldsymbol{\beta} \end{pmatrix} = \begin{pmatrix} \boldsymbol{g} \\ 0 \end{pmatrix} \quad M_{ij} = \varphi\left(\boldsymbol{x}_{k_i} - \boldsymbol{x}_{k_j}\right) \quad 1 \le i, j \le N \quad \mathbf{P} = \begin{bmatrix} 1 & x_{k_1} & y_{k_1} & z_{k_1} \\ 1 & x_{k_2} & y_{k_2} & z_{k_2} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{k_N} & y_{k_N} & z_{k_N} \end{bmatrix}$$

RBF Background

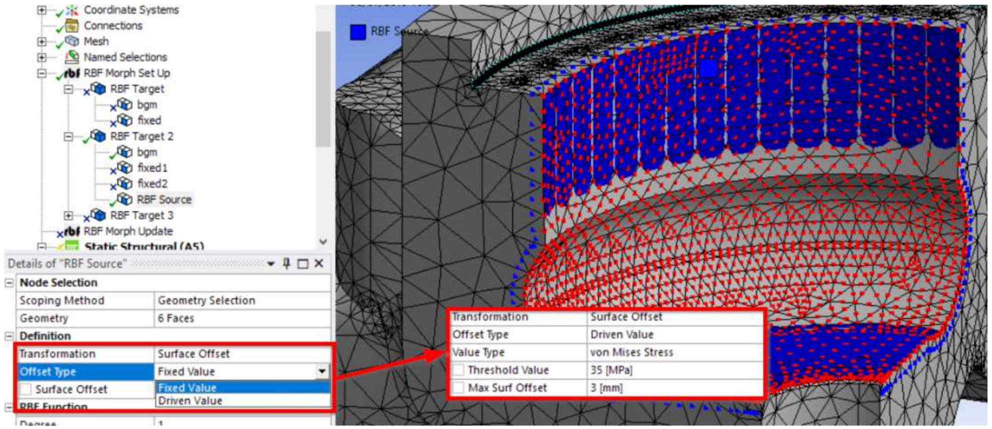
- Once solved the RBF problem each displacement component is interpolated:

$$\begin{cases} s_x(\mathbf{x}) = \sum_{i=1}^{N} \gamma_i^x \varphi\left(\mathbf{x} - \mathbf{x}_{k_i}\right) + \beta_1^x + \beta_2^x x + \beta_3^x y + \beta_4^x z \\ s_y(\mathbf{x}) = \sum_{i=1}^{N} \gamma_i^y \varphi\left(\mathbf{x} - \mathbf{x}_{k_i}\right) + \beta_1^y + \beta_2^y x + \beta_3^y y + \beta_4^y z \\ s_z(\mathbf{x}) = \sum_{i=1}^{N} \gamma_i^z \varphi\left(\mathbf{x} - \mathbf{x}_{k_i}\right) + \beta_1^z + \beta_2^z x + \beta_3^z y + \beta_4^z z \end{cases}$$

- Several different radial function (kernel) can be employed:

RBF	**φ(r)**	**RBF**	**φ(r)**
Spline type (Rn)	r^n, *n odd*	Inverse multiquadratic (IMQ)	$\frac{1}{\sqrt{1+r^2}}$
Thin plate spline	$r^n \log(r)$ *n even*	Inverse quadratic (IQ)	$\frac{1}{1+r^2}$
Multiquadratic (MQ)	$\sqrt{1+r^2}$	Gaussian (GS)	e^{-r^2}

- Automatic optimization is accoplished connecting adjoint and BGM data from numerical simulation to mesh morphing tool.
- Offset Surface shape modification allow to define for each node a displacement according to the local normal direction.
- When using BGM data, the direction and amplitude of displacement is defined according to BGM stress data, considering the threshold stress value σ_{th} and the d maximum displacement.

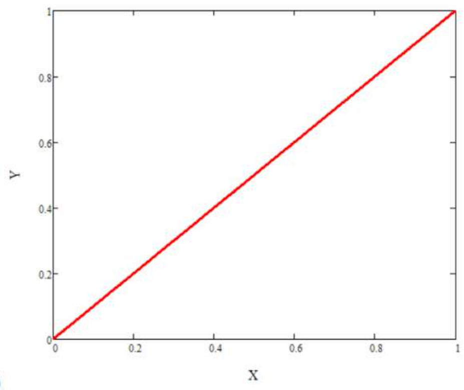


$$S_{node} = \frac{\sigma_{node} - \sigma_{th}}{\sigma_{max} - \sigma_{min}} \cdot d$$

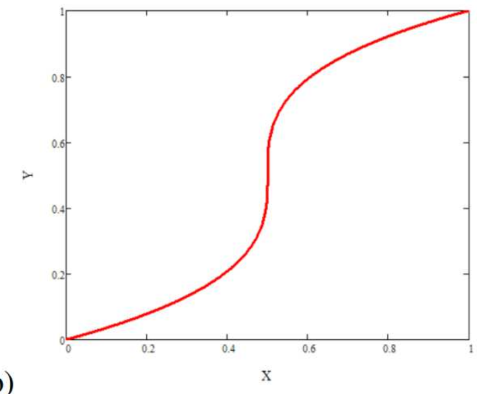
Automatic Surface Sculpting

- With adjoint approach, data from Topological optimization is used: for each node a topological density (which ranges between 0 and 1) function is defined to decide if a node has to be maintained or removed.
- This data can be interpolated using linear or irrational function to define a displacement field to be used by RBF Morph tool.
- Using the irrational function $S_{node\ adj}$ value is the displacement of each node, ρ is the interpolated topological density and d is the user defined maximum displacement.

$$S_{node\ adj} = \left[0.5\left(\frac{(\rho-0.5)}{0.5}\right)^{\frac{1}{3}} + 0.5\right]\cdot d, \quad with\ \rho \in [0, 1]$$



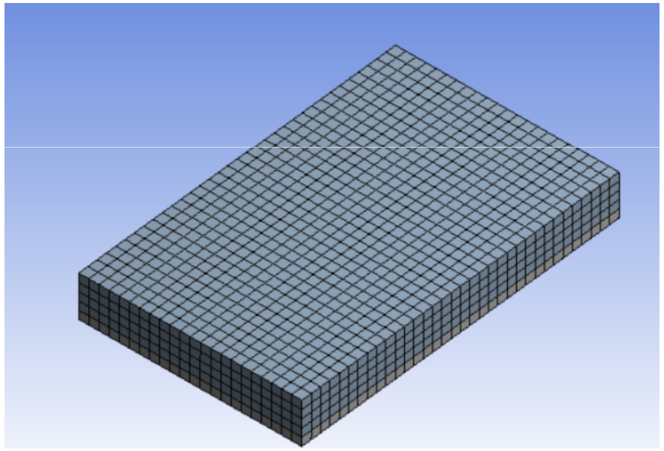
a)



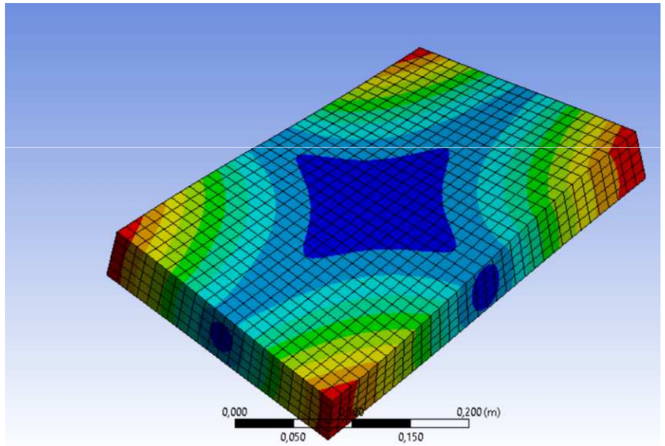
b)

Applications and Results – Simple Thick Plate – Adjoint

- Free and undamped dynamic analysis of a thick plate
- Optimized using adjoint approach to decrease mass and maintain first frequency above 1220 Hz

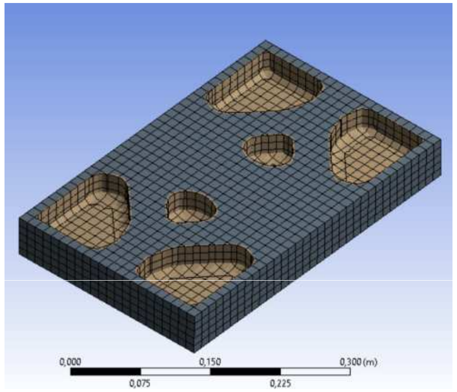


Mode	Freq. (Hz)
1	1457.3
2	1542.3
3	3159.1
4	3799.8
5	3816.2

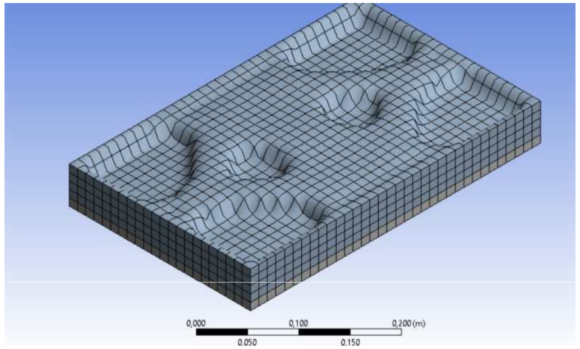


Applications and Results – Simple Thick Plate – Adjoint

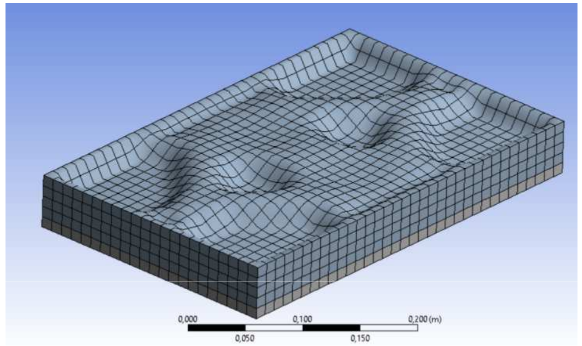
- Results ($d = 15mm$):



Topological Optimization

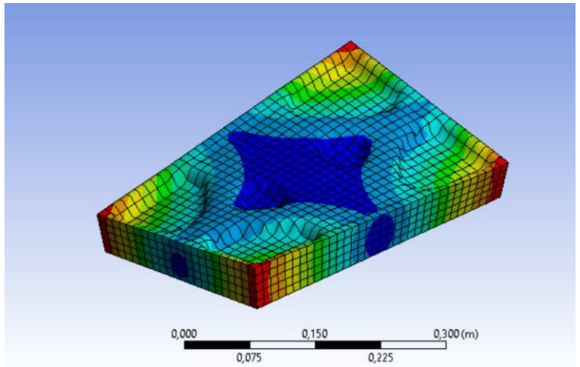


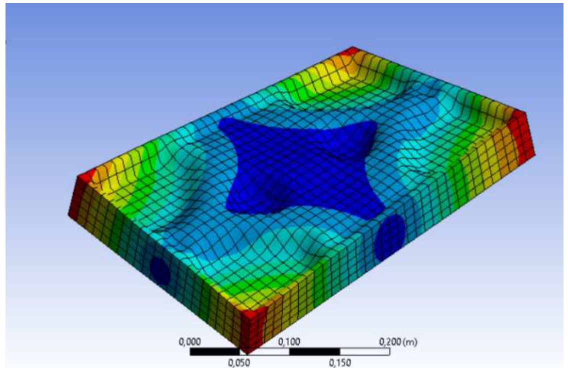
Irrational Interpolation



Linear Interpolation

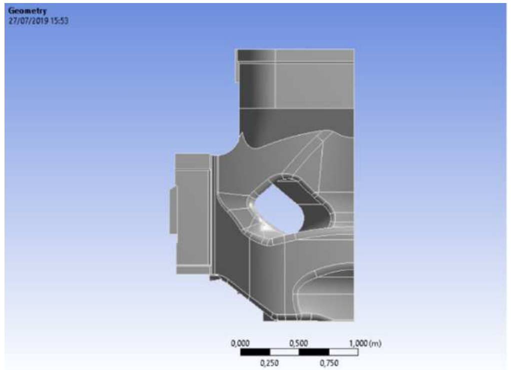
Mode	Baseline Freq. (Hz)	Linear int. Freq. (Hz)	Irrational int. Freq. (Hz)
1	1457.3	1315.6	1351.5
2	1542.3	1402.2	1440.7
…	…	…	…

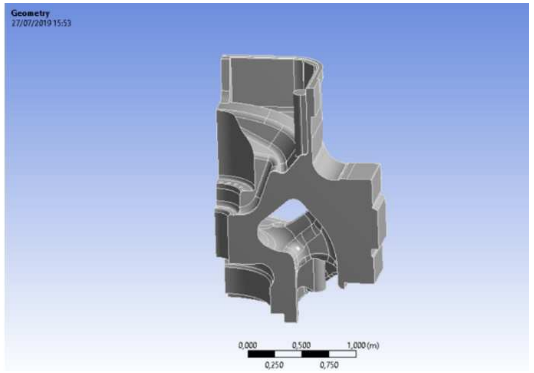




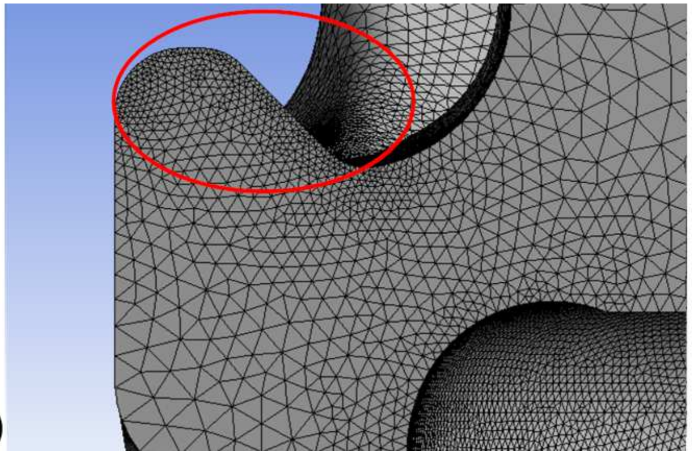
Applications and Results – Industrial Component – Adjoint

- Confidential Data
- Maximum reference stress reduction: -11.7 %

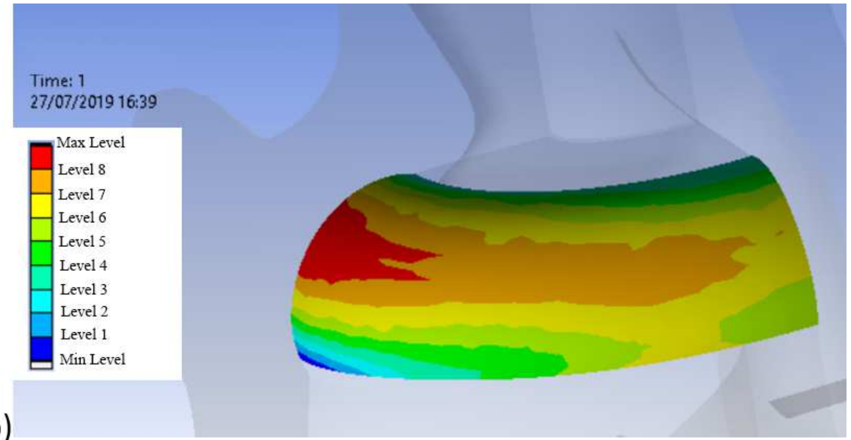




a)



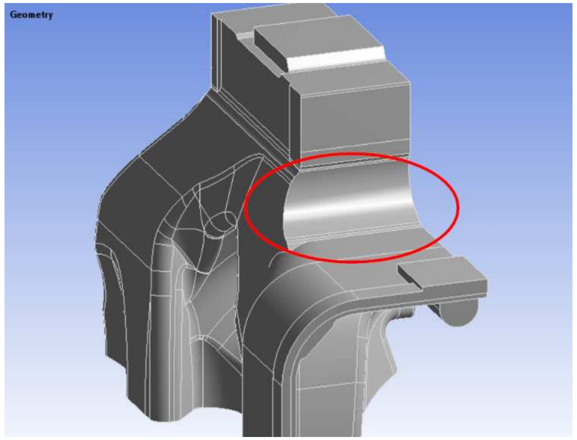
b)



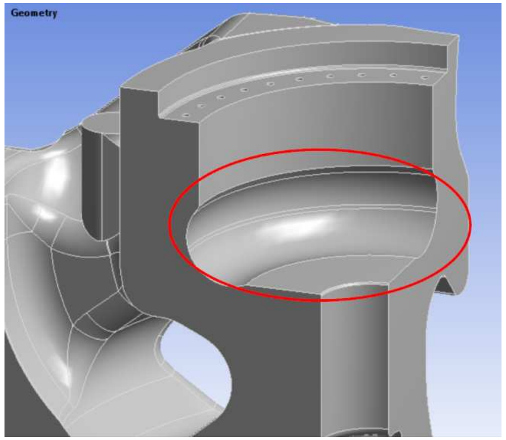
Applications and Results – Industrial Component – BGM

- Confidential Data
- 3 Hot Spots identified

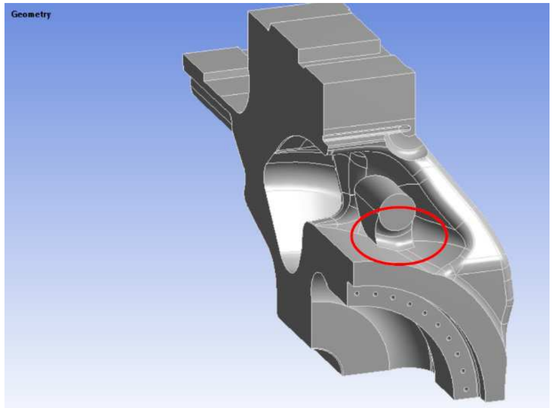
Location	σ_{th}	$d\ [mm]$
Hot Spot 1	1457.3	3
Hot Spot 2	1542.3	3
Hot Spot 3	3159.1	1



Hot Spot 1



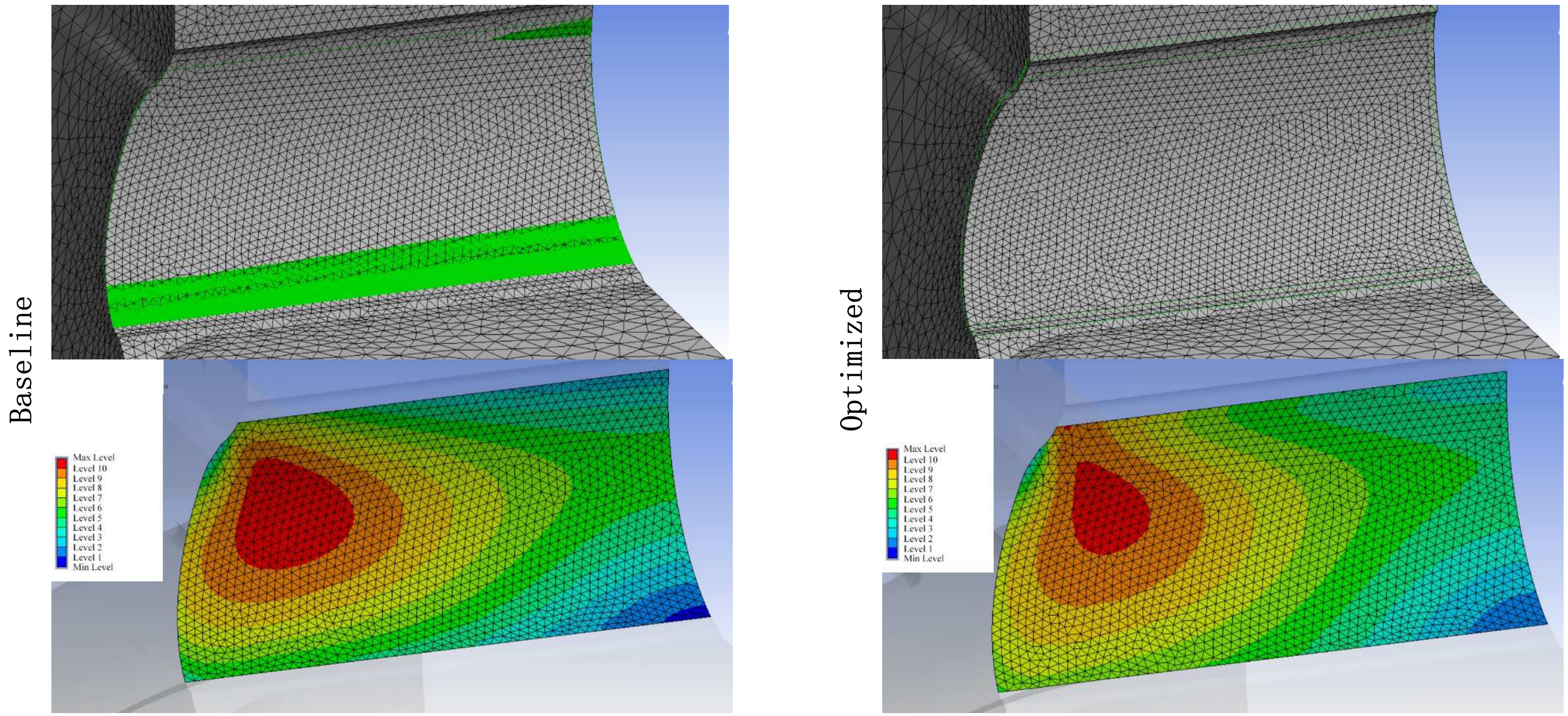
Hot Spot 2



Hot Spot 3

Applications and Results – Industrial Component – BGM

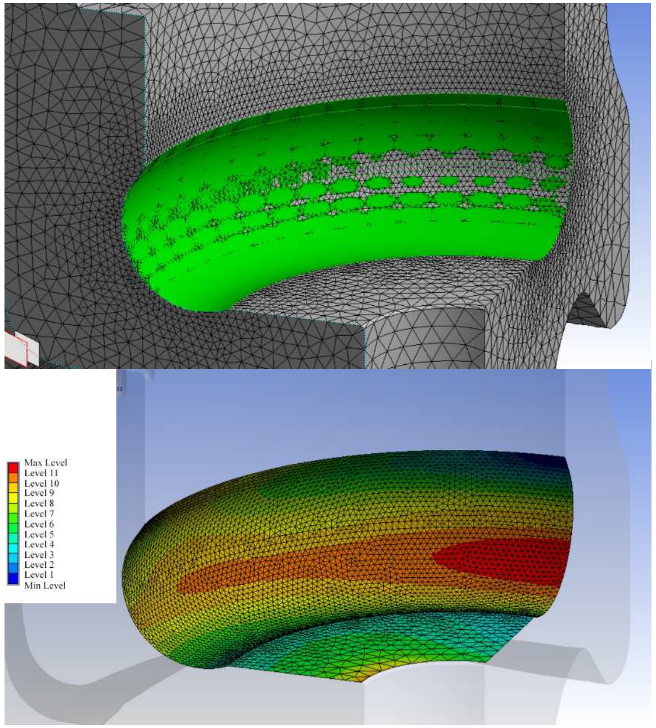
- Hot Spot 1
- Max stress reduction: 2.76%



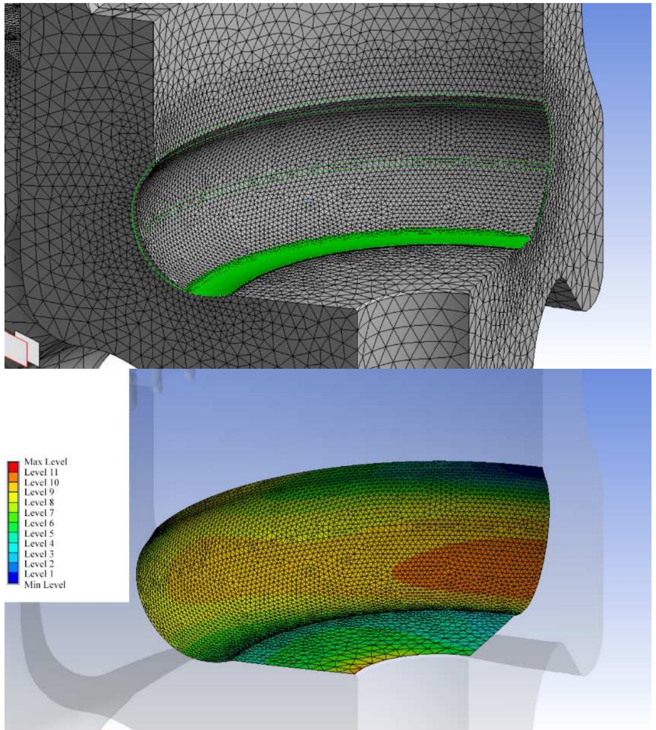
Applications and Results – Industrial Component – BGM

- Hot Spot 2
- Max stress reduction: 7.85%

Baseline



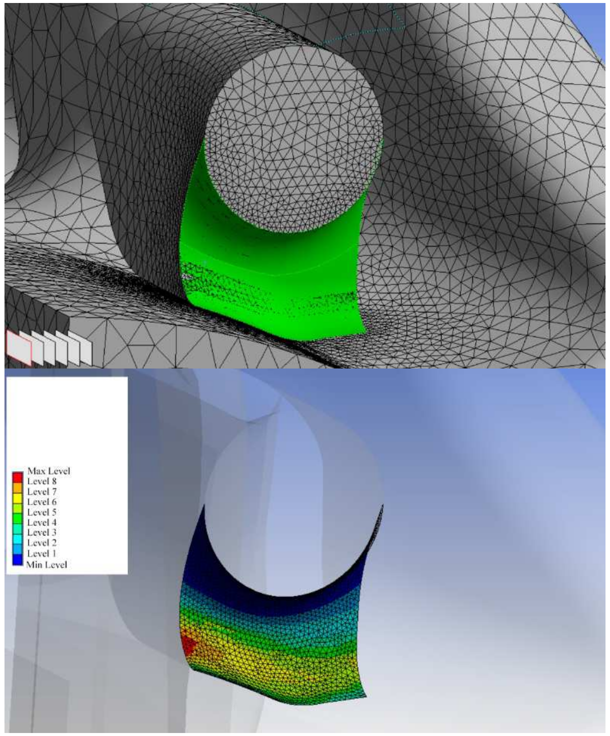
Optimized



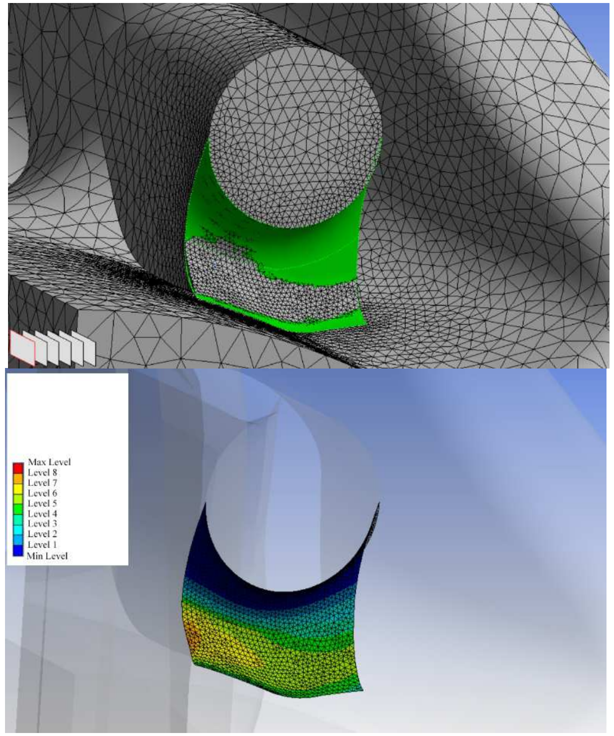
Applications and Results – Industrial Component – BGM

- Hot Spot 3
- Max stress reduction: 8.12%

Baseline



Optimized



Conclusions

- A **methodology** to perform **automatic shape optimization via surface sculpting** was presented.
- The methodology was developed using **Ansys Workbench** and the **RBF Morph** ACT extensions.
- Two approaches were investigated **Biological Growth Method** and **Adjoint**, which were successfully coupled with mesh morphing tool.
- **Adjoint** method, at the cost of an additional computation, gives sensitivities with respect of objective functions which were used to identify surface nodes to be moved inward or outward.

Conclusions

- **BGM** uses surfaces stress levels to identify model zones to be moved inward or outward, obtaining stress peak minimization and more uniform stress distribution.
- Both approaches were illustrated on **simple geometries** and then applied to a more **complex case.**
- The proposed procedure allowed to reach **optimised shapes** matching different **objective functions** in an automatic way, with a **minimal effort from the user.**
- The framework adopted was composed by ANSYS Workbench with RBF Morph ACT extension, which provided an integrated tool to the user in which perform optimisation task

Thank You For Your Kind Attention!

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[b]Sacmi Imola S.C., Via Prov.le Selice 17/a, Imola 40026, Ital

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