



# Automatic shape optimization of structural components with manufacturing constraints

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# Outline

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- Introduction
- RBF Background
- BGM Background
- Challenges
- Applications Description
- Optimization Results
- Conclusions

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- Mechanical component **optimization** is a paramount target in every engineering application.
  - A valuable tool for optimization in complex load and constraint configuration is the **Finite Element Method (FEM)**, which allows to test **different configurations** before the prototyping phase.
  - Optimization strategies are often based on **parametrization** of the FEM model: the optimal configuration is found among a family of configurations obtained **varying** the **parameters** describing the model geometry.
  - Another possible optimization strategy exploits the results coming from FEM: **Biological Growth Method (BGM)** derives the component shape modification analysing the surface stress levels.

- Both procedures, parameter based and BGM, require the **generation of additional FEM models**: this task can be very **time-consuming** specially dealing with complex shape components.
- To overcome this problem **Mesh morphing** can be adopted: it allows to generate new FEM models without modifying the geometry and without remesh it.
- Furthermore, in conjunction with the BGM approach, thanks to mesh morphing a **high grade of automation** can be achieved.
- In the present work, the tool adopted for morphing the **FEM** mesh is **RBF Morph™**, which is based on Radial Basis Functions (**RBFs**).

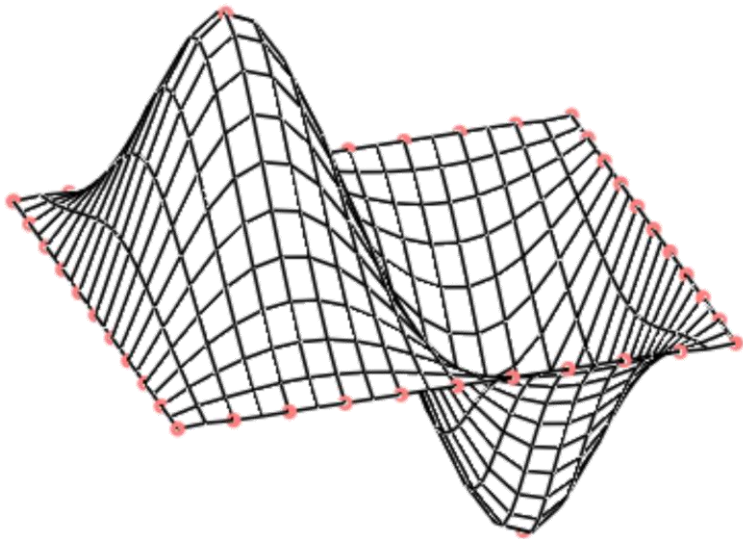
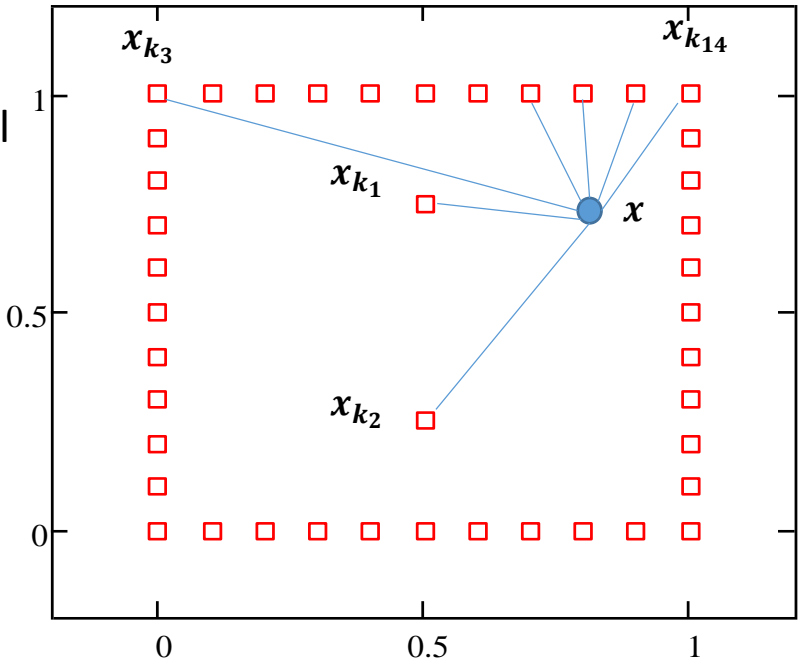
**(rbf-morph)™**

# RBF Background

- RBFs are a mathematical tool capable to **interpolate** in a generic point in the space a function **known** in a discrete set of points (**source points**).
- The interpolating function is composed by a **radial basis** and by a **polynomial**:

$$s(\mathbf{x}) = \sum_{i=1}^N \underbrace{\gamma_i \varphi(\|\mathbf{x} - \mathbf{x}_{k_i}\|)}_{\text{radial basis}} + \underbrace{h(\mathbf{x})}_{\text{polynomial}}$$

distance from the i-th source point



# RBF Background

- If evaluated on the source points, the interpolating function gives exactly the input values:

$$\begin{aligned} s(\mathbf{x}_{k_i}) &= g_i \\ h(\mathbf{x}_{k_i}) &= 0 \end{aligned} \quad 1 \leq i \leq N$$

- The RBF problem (evaluation of coefficients  $\boldsymbol{\gamma}$  and  $\boldsymbol{\beta}$ ) is associated to the solution of the linear system, in which  $\mathbf{M}$  is the interpolation matrix,  $\mathbf{P}$  is a constraint matrix,  $\mathbf{g}$  is the vector of known values on the source points:

$$\begin{bmatrix} \mathbf{M} & \mathbf{P} \\ \mathbf{P}^T & \mathbf{0} \end{bmatrix} \begin{pmatrix} \boldsymbol{\gamma} \\ \boldsymbol{\beta} \end{pmatrix} = \begin{pmatrix} \mathbf{g} \\ \mathbf{0} \end{pmatrix} \quad M_{ij} = \varphi(\mathbf{x}_{k_i} - \mathbf{x}_{k_j}) \quad 1 \leq i, j \leq N \quad \mathbf{P} = \begin{bmatrix} 1 & x_{k_1} & y_{k_1} & z_{k_1} \\ 1 & x_{k_2} & y_{k_2} & z_{k_2} \\ \mathbf{M} & \mathbf{M} & \mathbf{M} & \mathbf{M} \\ 1 & x_{k_N} & y_{k_N} & z_{k_N} \end{bmatrix}$$

- Once solved the RBF problem each displacement component is interpolated:

$$\begin{cases} s_x(\mathbf{x}) = \sum_{i=1}^N \gamma_i^x \varphi(\mathbf{x} - \mathbf{x}_{k_i}) + \beta_1^x + \beta_2^x x + \beta_3^x y + \beta_4^x z \\ s_y(\mathbf{x}) = \sum_{i=1}^N \gamma_i^y \varphi(\mathbf{x} - \mathbf{x}_{k_i}) + \beta_1^y + \beta_2^y x + \beta_3^y y + \beta_4^y z \\ s_z(\mathbf{x}) = \sum_{i=1}^N \gamma_i^z \varphi(\mathbf{x} - \mathbf{x}_{k_i}) + \beta_1^z + \beta_2^z x + \beta_3^z y + \beta_4^z z \end{cases}$$

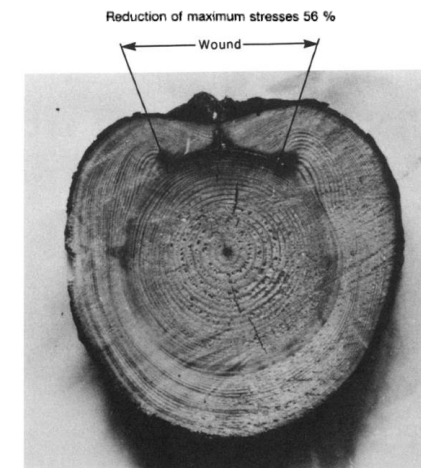
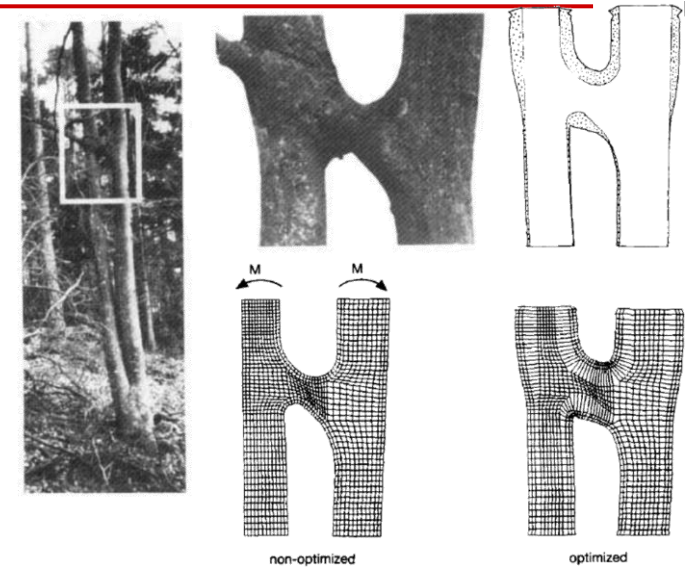
- Several different radial function (kernel) can be employed:

RBF	$\varphi(r)$	RBF	$\varphi(r)$
Spline type (Rn)	$r^n, n \text{ odd}$	Inverse multiquadratic (IMQ)	$\frac{1}{\sqrt{1+r^2}}$
Thin plate spline	$r^n \log(r) \ n \text{ even}$	Inverse quadratic (IQ)	$\frac{1}{1+r^2}$
Multiquadratic (MQ)	$\sqrt{1+r^2}$	Gaussian (GS)	$e^{-r^2}$

# BGM Background



- **BGM** approach is based on the observation that **biological** structures growth is driven by **local** level of **stress**.
- Bones and trees' trunks are able to **adapt the shape** to mitigate the stress level due to external loads.
- The process is driven by stress **value at surfaces**. Material can be **added or removed** according to local values.
- Was proposed by Mattheck & Burkhardt in 1990\*



\*Mattheck C., Burkhardt S., 1990. A new method of structural shape optimization based on biological growth. *Int. J. Fatigue* 12(3):185-190.



- The BGM idea is that surface growth can be expressed as a **linear law** with respect to a given threshold value:

$$\dot{\varepsilon} = k (\sigma_{Mises} - \sigma_{ref})$$

- Waldman and Heller\* refined this first approach proposing a **multi peak** one:

$$d_i^j = \left( \frac{\sigma_i^j - \sigma_i^{th}}{\sigma_i^{th}} \right) \cdot s \cdot c, \quad \sigma_i^{th} = \max(\sigma_i^j) \text{ if } \sigma_i^j > 0 \quad \text{or} \quad \sigma_i^{th} = \min(\sigma_i^j) \text{ if } \sigma_i^j < 0$$

- In **RBF Morph ANSYS Workbench ACT extension** a different implementation is present and different **stress types** can be used to modify the surface shape:

$$S_{node} = \frac{\sigma_{node} - \sigma_{th}}{\sigma_{max} - \sigma_{min}} \cdot d$$

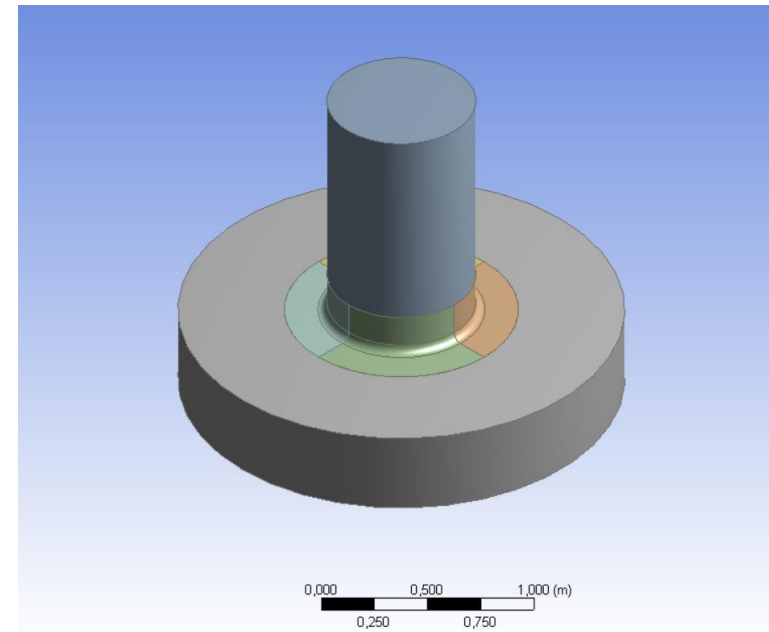
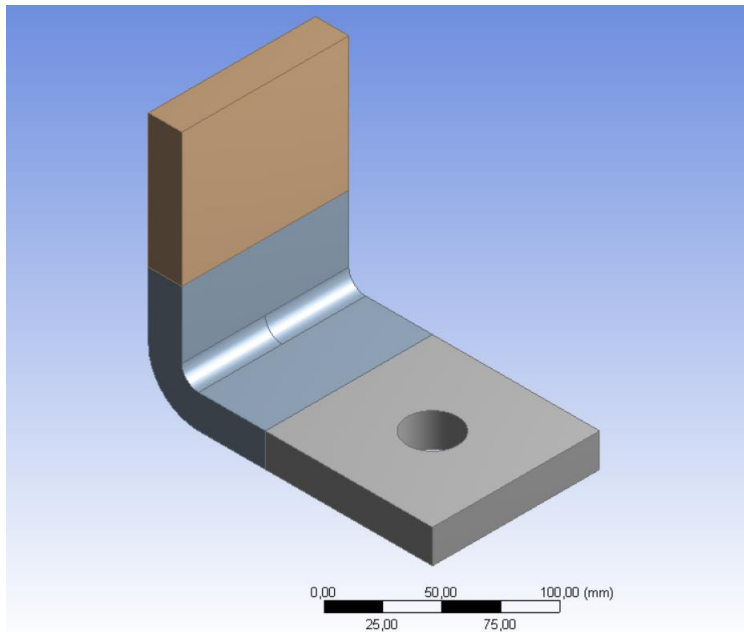
Stress/strain type	Equation	Stress/strain type	Equation
von Mises stress	$\sigma_e = \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$	Stress intensity	$\sigma_e = \max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ,  \sigma_3 - \sigma_1 )$
Maximum principal stress	$\sigma_e = \max(\sigma_1, \sigma_2, \sigma_3)$	Maximum Shear stress	$\sigma_e = 0.5 \cdot (\max(\sigma_1, \sigma_2, \sigma_3) - \min(\sigma_1, \sigma_2, \sigma_3))$
Minimum principal stress	$\sigma_e = \min(\sigma_1, \sigma_2, \sigma_3)$	Eqv. plastic strain	$\varepsilon_e = [2(1 + \nu')]^{-1} \cdot \left( 0.5 \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2} \right)$

\*Waldman W., Heller M., 2015. Shape optimization of holes in loaded plates by minimization of multiple stress peaks, Defence Science and Technology Organisation Fisherman Bend, Australia, Aerospace Div, <http://www.dtic.mil/docs/citations/ADA618562>.

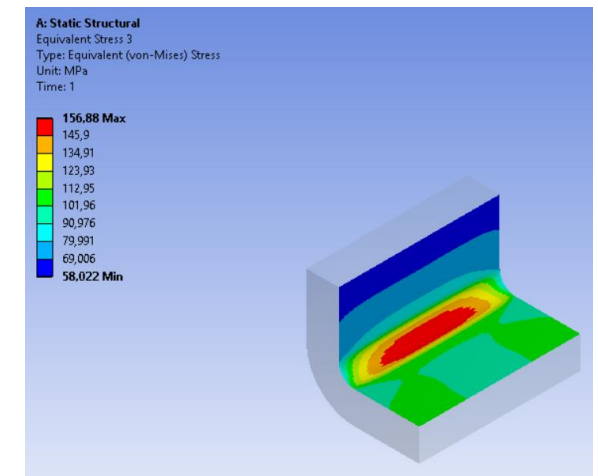
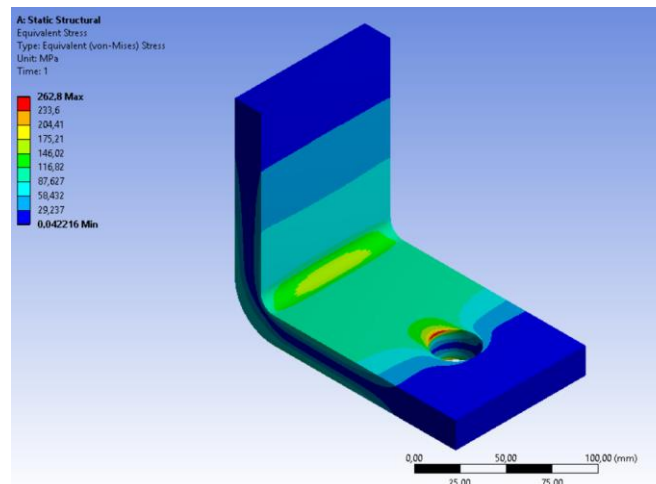
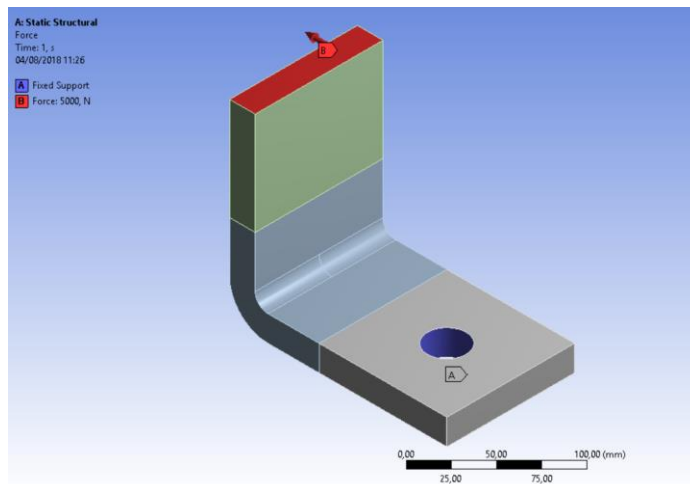
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- Currently the mesh morphing allows to obtain complex shape modifications without remeshing, but can require a **lot of efforts** in order to maintain specific manufacturing constraints.
  - In several industrial application the capability of replicate the shape modification along a direction or around an axis is a **strong requirement**.
  - RBF Morph ACT extension introduced in the last version a new feature in order to satisfy these requirements.
  - The **Coordinate Filtering feature** allows the user to replicate a specific RBF solution (i.e. shape modification) **along** or **around** a specified **axis**.

# Applications Description

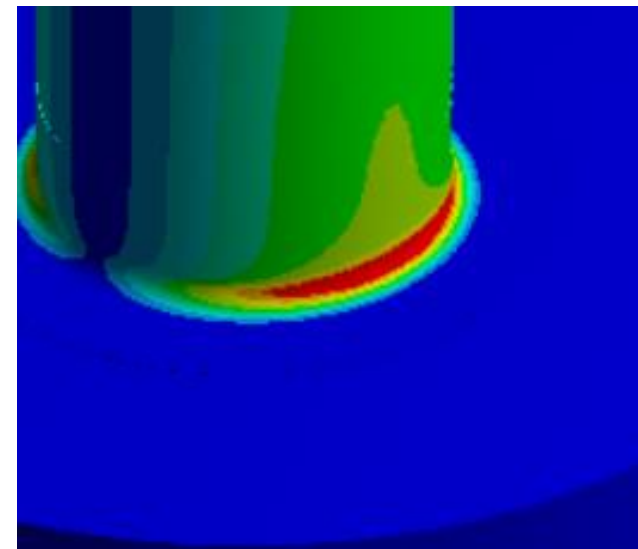
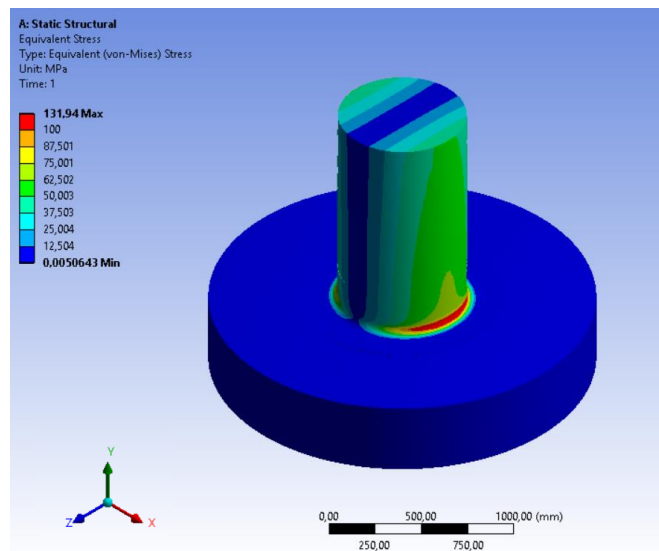
- To demonstrate the effectiveness of the Coordinate Filtering feature two applications were developed: a first one to apply a **linear manufacturing constraint** and a second one to apply a **circular manufacturing constraint**.



- The bracket was constrained at the hole and loaded at the upper surface.
- Von Mises stress hot spots are located at hole and at fillet. The latter one will be the target of the optimization.
- Maximum von Mises stress at fillet in baseline configuration is 156 MPa.



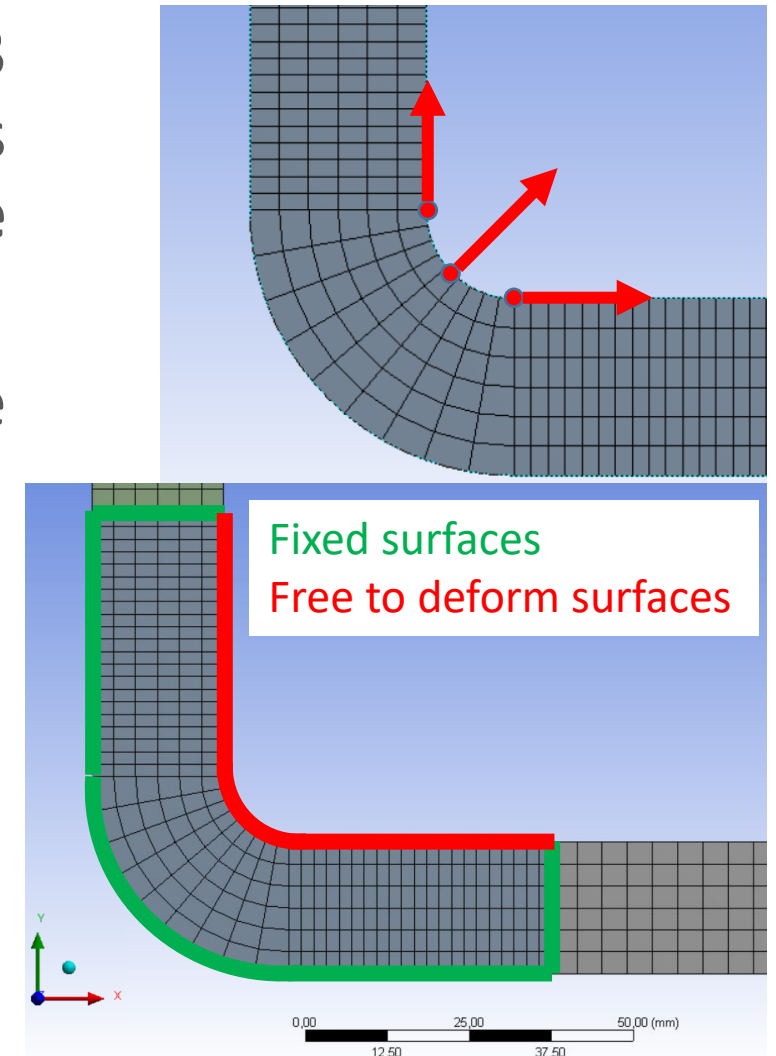
- The pin was constrained at the lower surface and loaded at the upper surface by means of a remote force.
- Von Mises stress hot spots is located at fillet and will be the target of the optimization.
- Maximum von Mises stress at fillet in the baseline configuration is 132 MPa.



# Results – linear manufacturing constraints – parameters

- Parameter based optimization was set up with **3 parameters**. Shape resulting from points displacement was replicated using **Coordinate Filtering**.
- **Ansys Design Xplorer** was employed to optimize shape using the **Response Surface Optimization**:

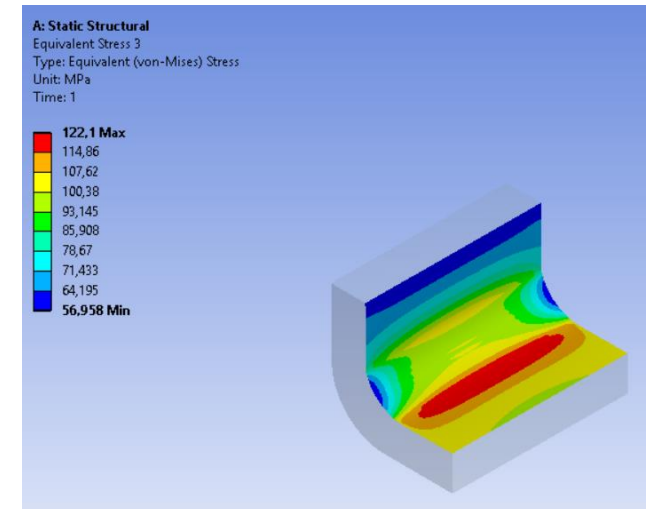
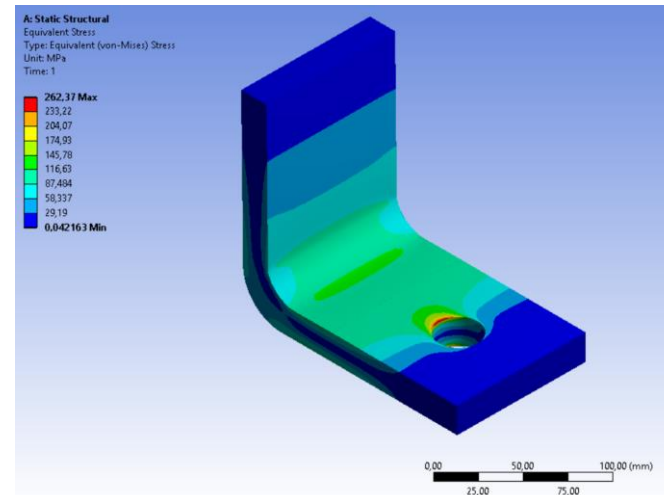
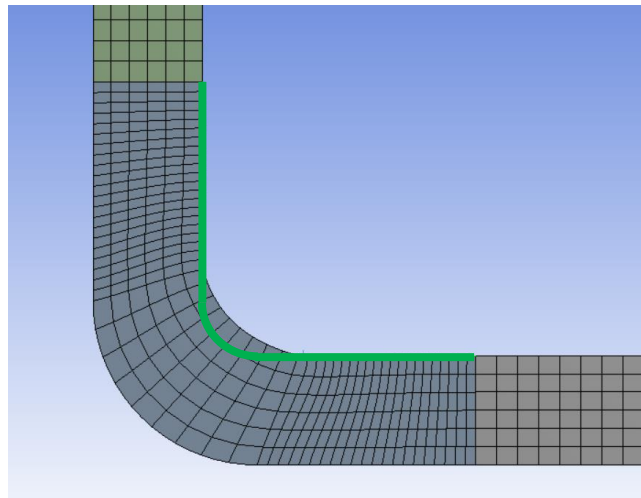
Design of Experiment type	Latin Hypercube
Samples type	CCD Samples
Response Surface type	Kriging
Kernel type	Variable
Refinement points	3 – candidate points



# Results – linear manufacturing constraints – parameters



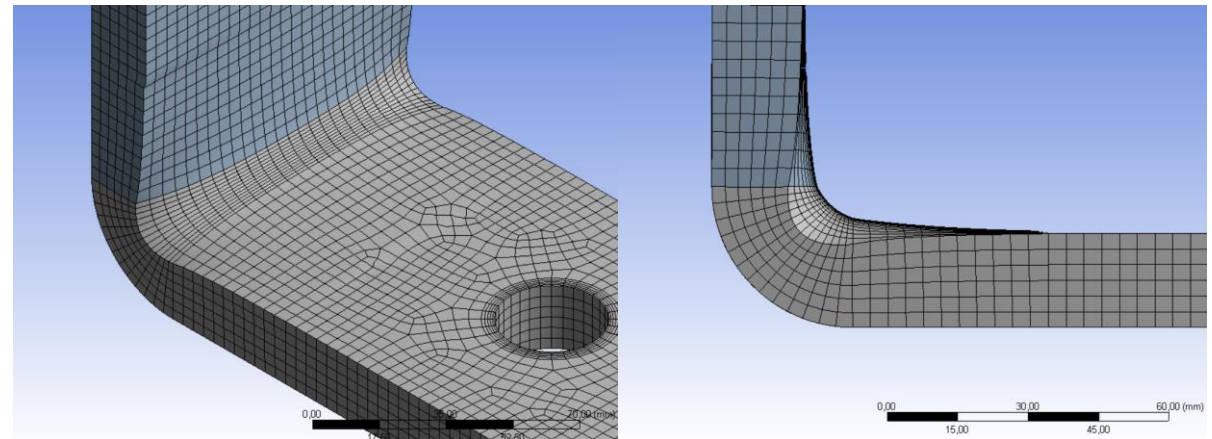
- With the optimized configuration obtained by Response Surface Optimization, the maximum von Mises stress value is 122 MPa, **reduced by 21.8%**.
- The optimized shape is compliant with linear manufacturing constraint even if it was obtained controlling only 3 points.



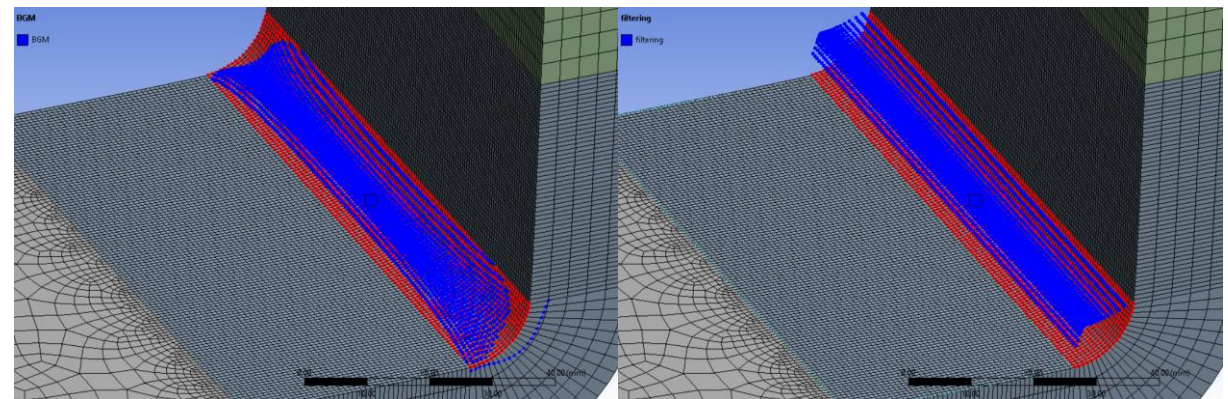


## Results – linear manufacturing constraints – BGM

- When using BGM final **shape** can be very **complex**.
- Coordinate Filtering is **required** if manufacturing constraints are required.



Not filtered BGM



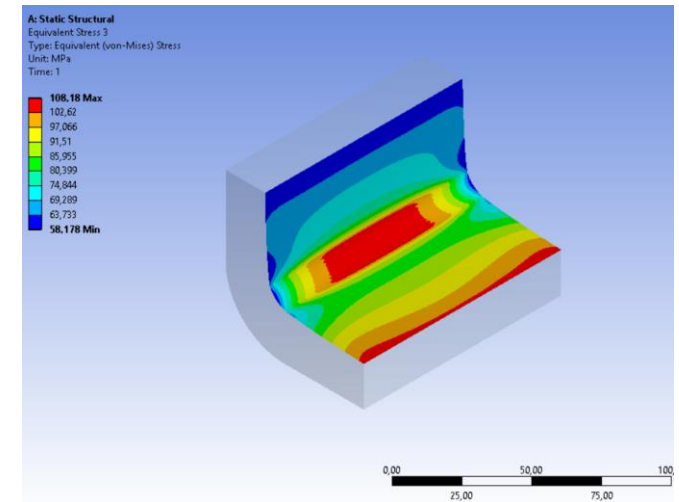
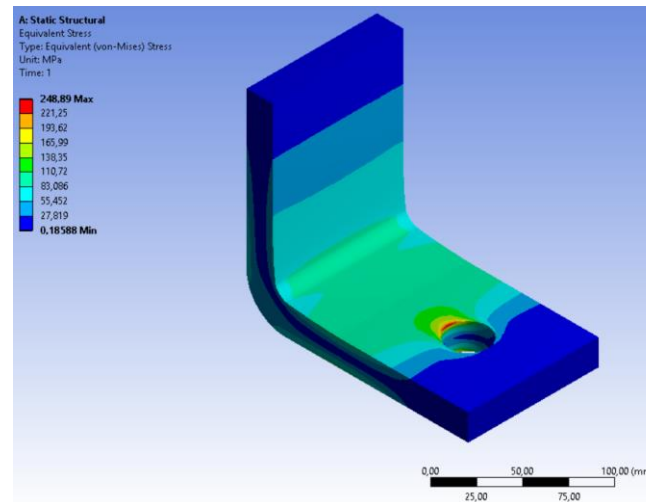
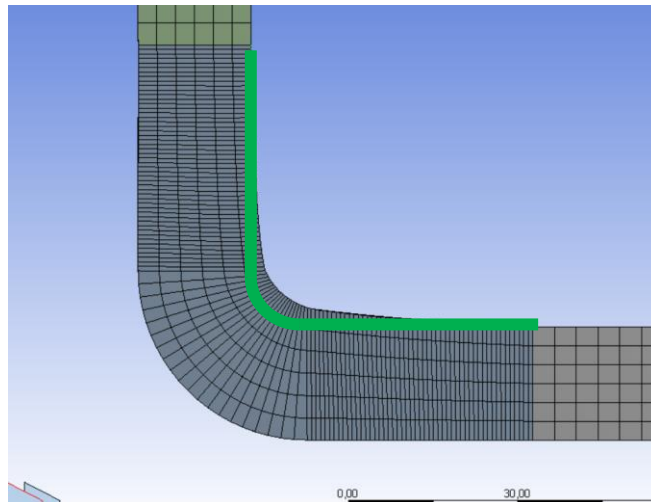
Not filtered vs. filtered BGM – amplified displacements



# Results – linear manufacturing constraints – BGM

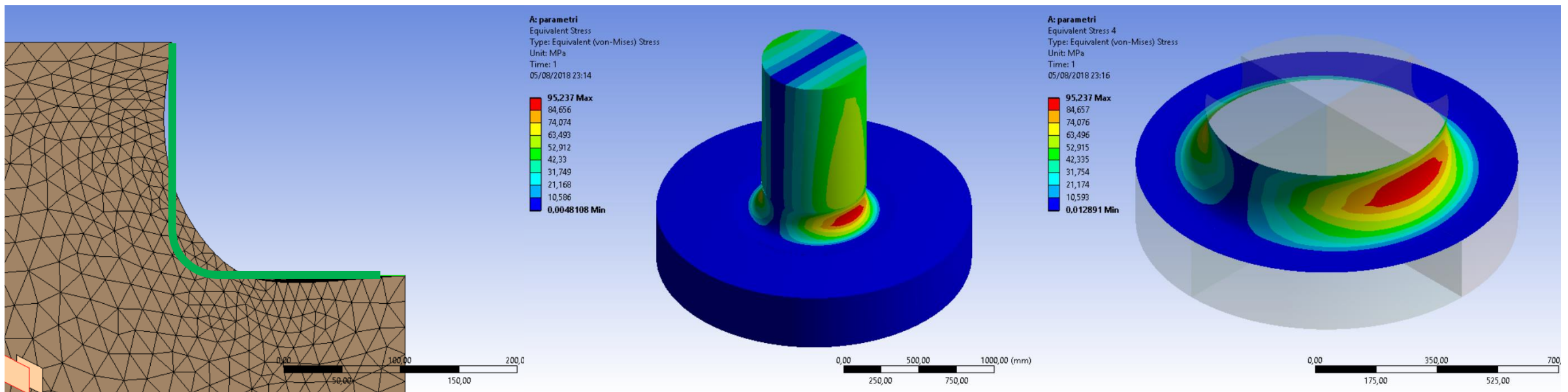


- **BGM** optimization was performed on the fillet surfaces using as **threshold** von Mises stress **100 MPa** and **maximum displacement 1 mm**. The BGM optimization was iterated **10 times**.
- With the optimized configuration obtained by BGM optimization, the maximum von Mises stress value is 108 MPa, **reduced by 30.7%**.



## Results – circular manufacturing constraints

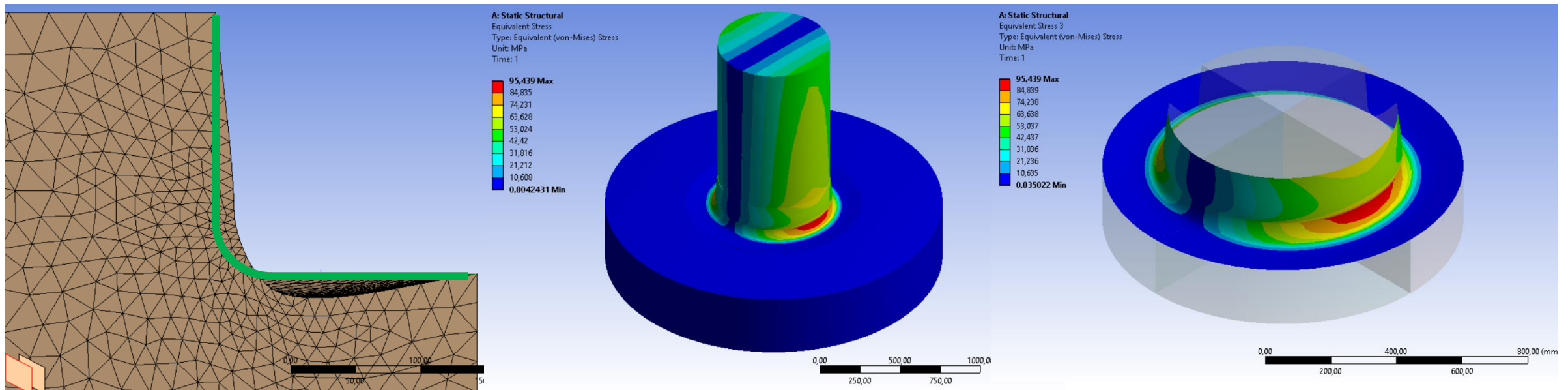
- The same **3-parameters** approach was applied to the pin model.
- In this case with the parameter based optimization the maximum von Mises stress in the fillet area is reduced to 95 MPa, a **reduction of 28%** with respect the baseline configuration.



## Results – circular manufacturing constraints



- **BGM** optimization was performed on the fillet surfaces using as **threshold** von Mises stress **75 MPa** and **maximum displacement 5 mm**. The BGM optimization was iterated **10 times**.
- With the optimized configuration obtained by BGM optimization, the maximum von Mises stress value is 95 MPa, **reduced by 28%**.



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- A **methodology** to obtain **optimized shape** suitable for traditional manufacturing processes **was developed**.
  - The methodology was developed using **Ansys Workbench** and the **RBF Morph ACT** extensions.
  - Optimization was performed using **BGM** and **parametric optimization** which, generally speaking, do not guarantee that linear or cyclic symmetry are respected.
  - It was demonstrated that with these tools the **linear and circular features** can be **preserved** in the optimized configuration.
  - Optimization was performed directly **controlling the shape** (parameter based) and **exploiting numerical results** regarding surface stresses (BGM).
  - With both approaches stress reduction was between the range of **21% - 30%**.
  - Proposed methodology can be successfully **adopted** and **implemented** in the design cycle of parts or components that are subjected to circular and linear manufacturing constraints.



Thank You For Your Kind Attention!



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