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Radial Basis Functions mesh morphing FOR the analysis of CRACKS propagation

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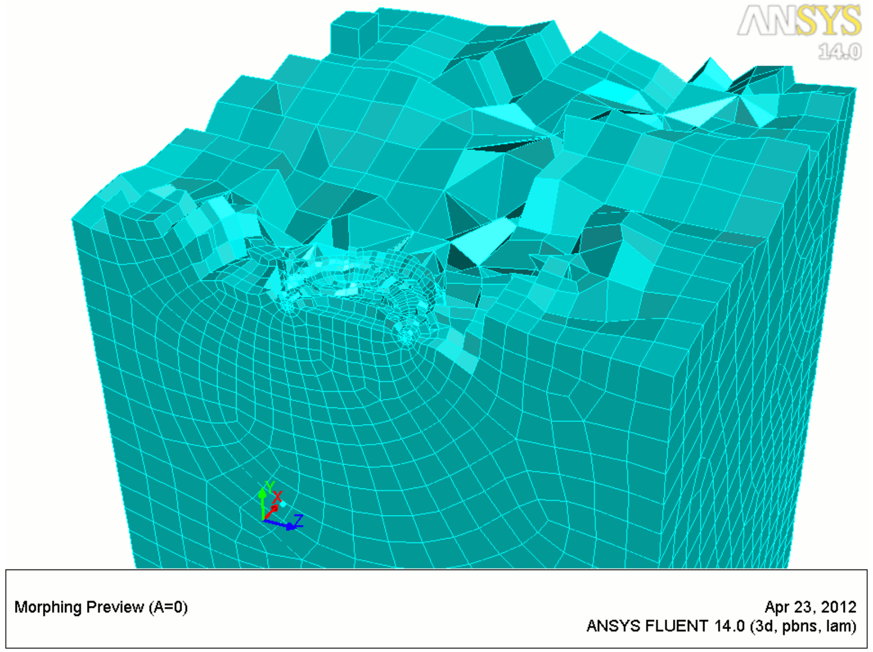
Introduction

- The fatigue life of a structural component copes with the initiation and propagation of a **crack**.
- The Stress-Intensity Factors (**SIFs**), which can be deducted via Finite Element Method (**FEM**) analyses, together with the Paris-Erdogan Law can be used to investigate the crack stability and growth.
- The update of the **FEM** mesh onto new crack shape can be performed by **re-meshing** (classic approach) which automation could be complex and painful.
- The new shape of the crack can be obtained applying mesh morphing to a baseline FEM model of the flawed part, thus **reducing** drastically the time needed to **generate** the new FEM model.
- In the present work, the tool adopted for morphing the **FEM** mesh is **RBF Morph™**, which is based on Radial Basis Functions (**RBFs**).

Research path

- The first **prototype** was defined in 2012 in cooperation with ANSYS R&D office in Lyon.
- Just **morphing concept** (a Fluent mesh with linear cells) successfully demonstrated.
- RBF Morph was introduced in ANSYS Mechanical in 2014.
- RBF4CRACKS project (2016 ends March 2018) was funded by the University of Rome Tor Vergata within the programme “**Consolidate the Foundations**”

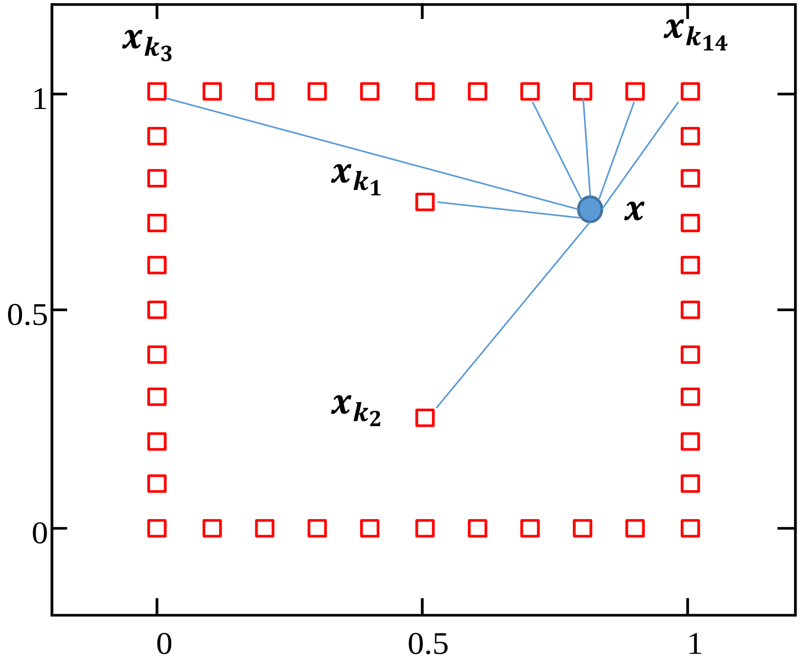


- RBFs are a mathematical tool capable to **interpolate** in a generic point in the space a function **known** in a discrete set of points (**source points**).
- The interpolating function is composed by a **radial basis** and by a **polynomial**:

radial basis — polynomial

$$s(\boldsymbol{x}) = \sum_{i=1}^{N} \gamma_i \varphi\left(\left\| \boldsymbol{x} - \boldsymbol{x}_{k_i} \right\|\right) + h(\boldsymbol{x})$$

distance from the i-th source point



RBF Background

- If evaluated on the source points, the interpolating function gives exactly the input values:

$$s(\boldsymbol{x}_{k_i}) = g_i \qquad 1 \le i \le N$$
$$h(\boldsymbol{x}_{k_i}) = 0$$

- The RBF problem (evaluation of coefficients γ and β) is associated to the solution of the linear system, in which **M** is the interpolation matrix, **P** is a constraint matrix, **g** is the vector of known values on the source points:

$$\begin{bmatrix} \mathbf{M} & \mathbf{P} \\ \mathbf{P}^{\mathrm{T}} & 0 \end{bmatrix} \begin{pmatrix} \boldsymbol{\gamma} \\ \boldsymbol{\beta} \end{pmatrix} = \begin{pmatrix} \boldsymbol{g} \\ 0 \end{pmatrix} \qquad M_{ij} = \varphi\left(\boldsymbol{x}_{k_i} - \boldsymbol{x}_{k_j}\right) \qquad 1 \le i, j \le N \qquad \mathbf{P} = \begin{bmatrix} 1 & x_{k_1} & y_{k_1} & z_{k_1} \\ 1 & x_{k_2} & y_{k_2} & z_{k_2} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{k_N} & y_{k_N} & z_{k_N} \end{bmatrix}$$

RBF Background

- Once solved the RBF problem each displacement component is interpolated:

$$
\begin{cases}
s_x(\boldsymbol{x}) = \sum_{i=1}^{N} \gamma_i^x \varphi\left(\boldsymbol{x} - \boldsymbol{x}_{\boldsymbol{k}_i}\right) + \beta_1^x + \beta_2^x x + \beta_3^x y + \beta_4^x z \\
s_y(\boldsymbol{x}) = \sum_{i=1}^{N} \gamma_i^y \varphi\left(\boldsymbol{x} - \boldsymbol{x}_{\boldsymbol{k}_i}\right) + \beta_1^y + \beta_2^y x + \beta_3^y y + \beta_4^y z \\
s_z(\boldsymbol{x}) = \sum_{i=1}^{N} \gamma_i^z \varphi\left(\boldsymbol{x} - \boldsymbol{x}_{\boldsymbol{k}_i}\right) + \beta_1^z + \beta_2^z x + \beta_3^z y + \beta_4^z z
\end{cases}
$$

- Several different radial function (kernel) can be employed:

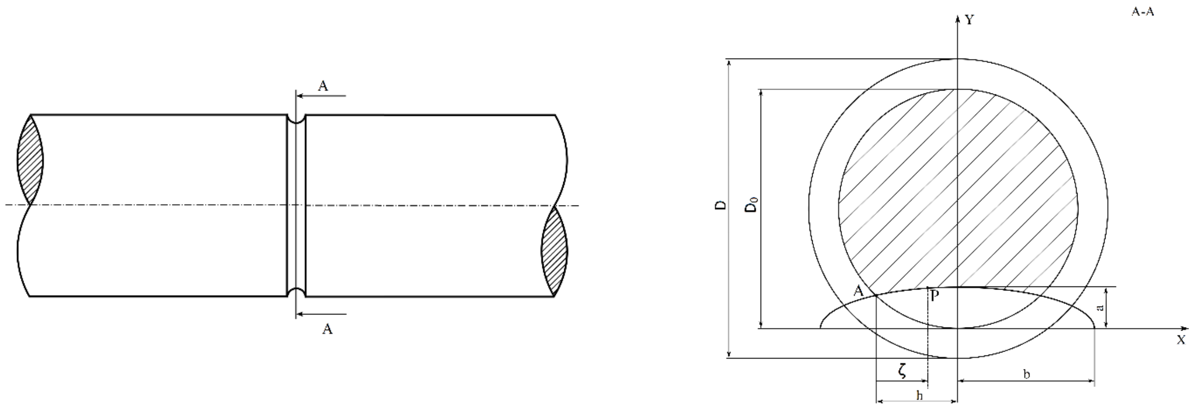
RBF	φ(r)	RBF	φ(r)
Spline type (Rn)	r^n, *n odd*	Inverse multiquadratic (IMQ)	$\frac{1}{\sqrt{1+r^2}}$
Thin plate spline	$r^n \log(r)$ *n even*	Inverse quadratic (IQ)	$\frac{1}{1+r^2}$
Multiquadratic (MQ)	$\sqrt{1+r^2}$	Gaussian (GS)	e^{-r^2}

Challenges

- The ability to **morph on a target crack shape** is highly required for maintenance inspections: a new (grown) shape of the crack is acquired and the FEA mesh needs to be updated.
- During **fatigue crack growth** the crack shape can evolve on a complex shape. Each growth step can be faced as a morphing action onto a **target crack shape**
- Preserving part shape (i.e. external surfaces at cracked hot spot) is a paramount. Advanced geometric modeler interaction and an **auxiliary CAD model** of the hot spot region could be used to support the analysis
- Mesh morphing produces element distortion (compression and stretching). The extent of crack shape variation without the generation of a new mesh is limited by **mesh quality**.

Application Description

- The approach was tested on a **round notched bar** with a constant curvature radius

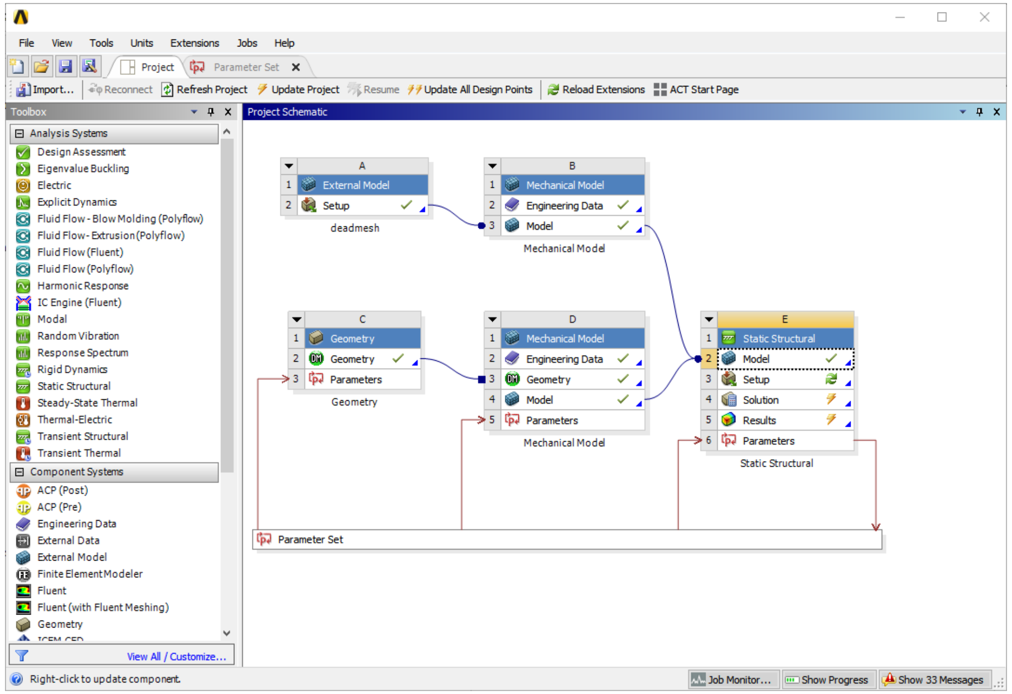


Parameter	Value
D (mm)	20
D_0 (mm)	16
L (mm)	80
ρ (mm)	2
F (kN)	30

- The workflow was completely build in **ANSYS® Workbench™**, adopting the proprietary **Fracture Tool** (FT) provided (midside node in quarter point position).
- The results from morphed meshes were compared with results from FT meshes.

Application Description

- The procedure was entirely developed into the **Workbenck** environment, thanks to the ACT extension RBF Morph.
- The mesh obtained with the ANSYS FT was added to the workflow as an external “dead” mesh. This ensures that the procedure can be successfully applied to a mesh obtained with any other mesh generation tool.



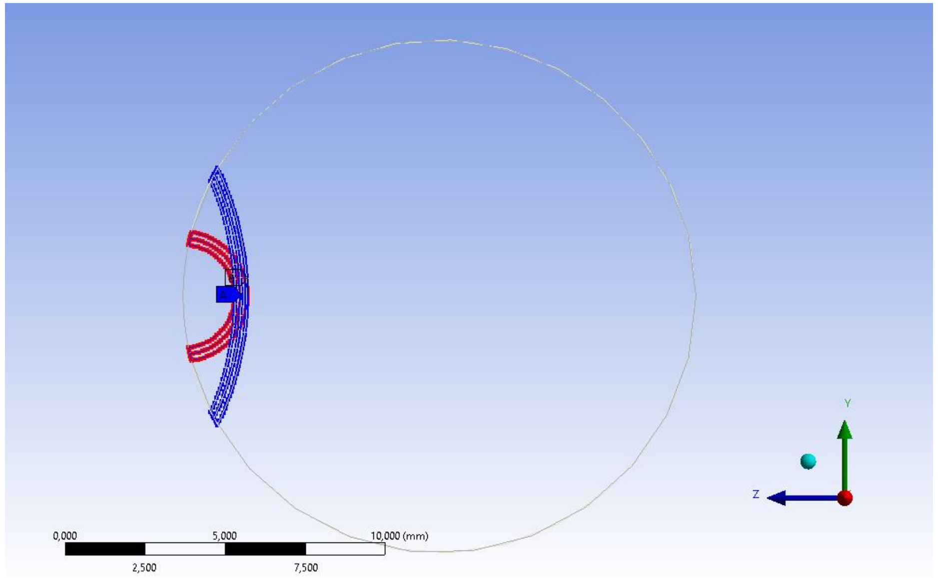
Result Assessment

- A three-dimensional model was realized using **10-node iso-parametric tetrahedrons** and **wedge** elements around the crack front.
- A preliminary convergence test was performed on the model without flaw to retrieve the stress concentration factor **Kt** (numerical value = **2.214**, theoretical* value = **2,2**); the final model is composed by **29K elements**.
- The baseline crack has a semi-elliptical shape with a = 1.6 mm and α=1
- A tensile load of **30 kN** was applied in a static analysis, pointwise SIFs along the crack correspond to $\mathbf{K_I}$.

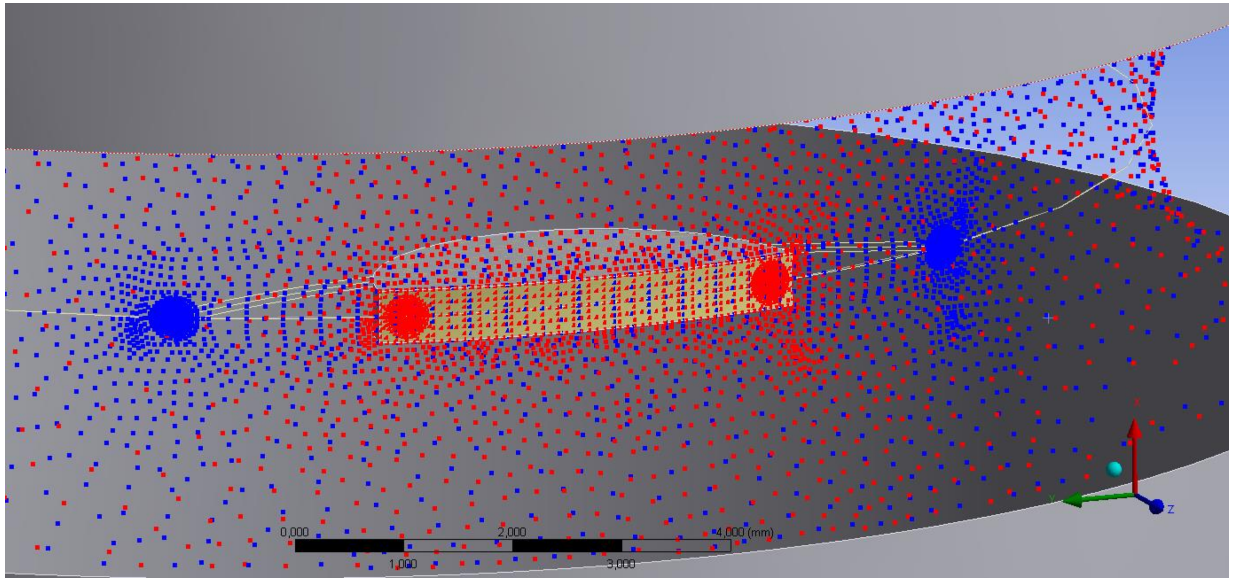
*Peterson's, Stress Concentration Factors, Second Edition, W. D. Pilkey

Result Assessment

- To preserve the wedge elements around the crack front, a particular set-up was adopted, involving three concentric curves which represent the crack front and the two traces of the tubular portion of the mesh around the crack.



auxiliary circle geometries



node preview of mesh morphing

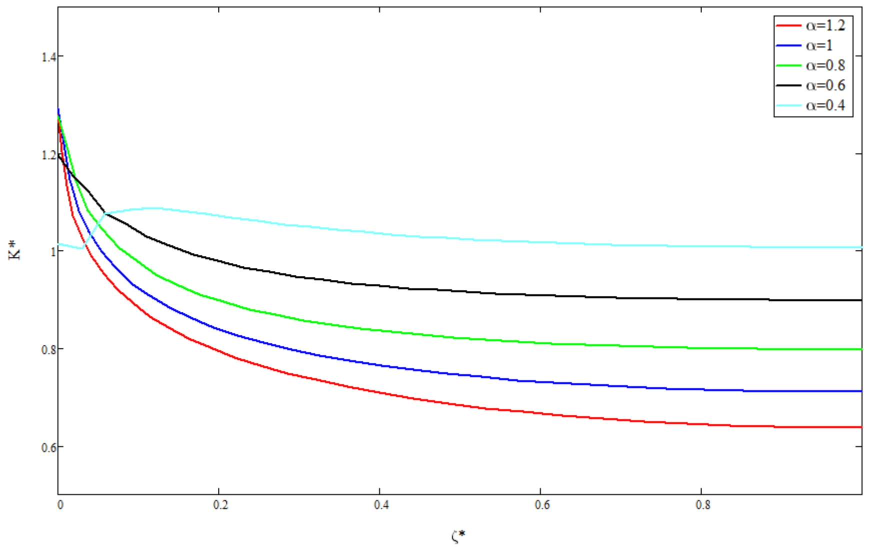
Result Assessment

- The results were **normalized** according to the following equations:

$$\zeta^* = \frac{\zeta}{h}$$ (Dimensionless curvilinear abscissa)

$$K_I^* = \frac{K_I}{\sigma_F \sqrt{\pi a}}$$ (Dimensionless SIF) where $$\sigma_F = \frac{2F}{\pi D_0^2}$$ (Nominal stress)

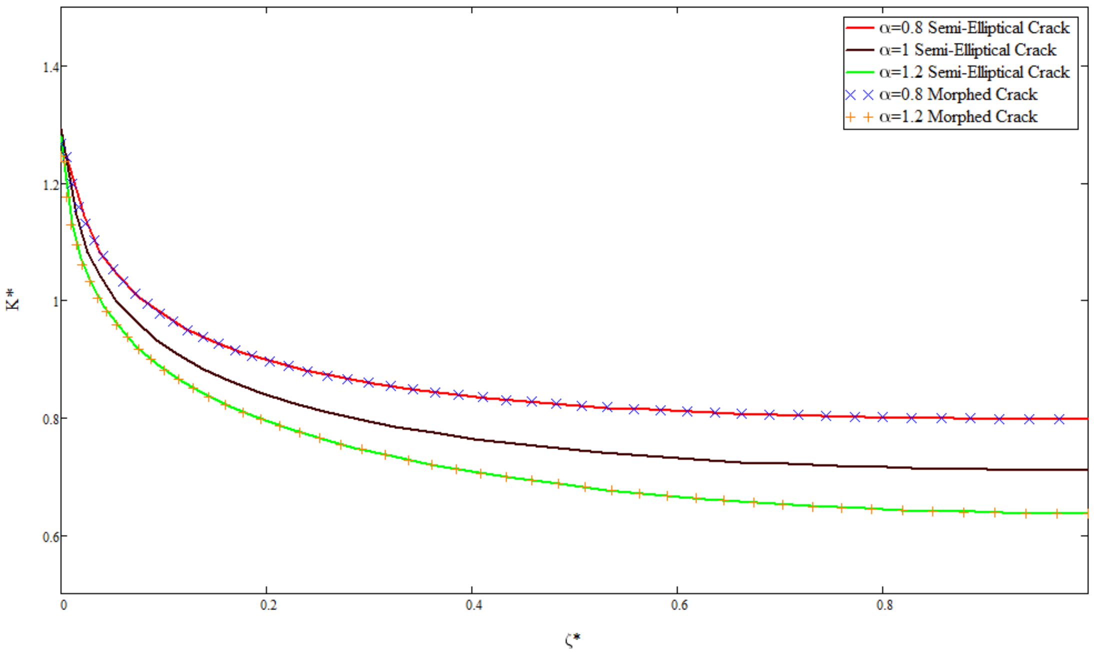
- A series of flaw geometries were realized using FT and compared with literature data obtaining a **good agreement**



Result Assessment

- The same crack geometries were obtained by means of **mesh morphing** starting from the baseline shape and results were compared with the results obtained from FT.

- A **perfect match** was achieved with the two methods.
- The morphed meshes, despite the deformation imposed, had still an **acceptable level of quality**.



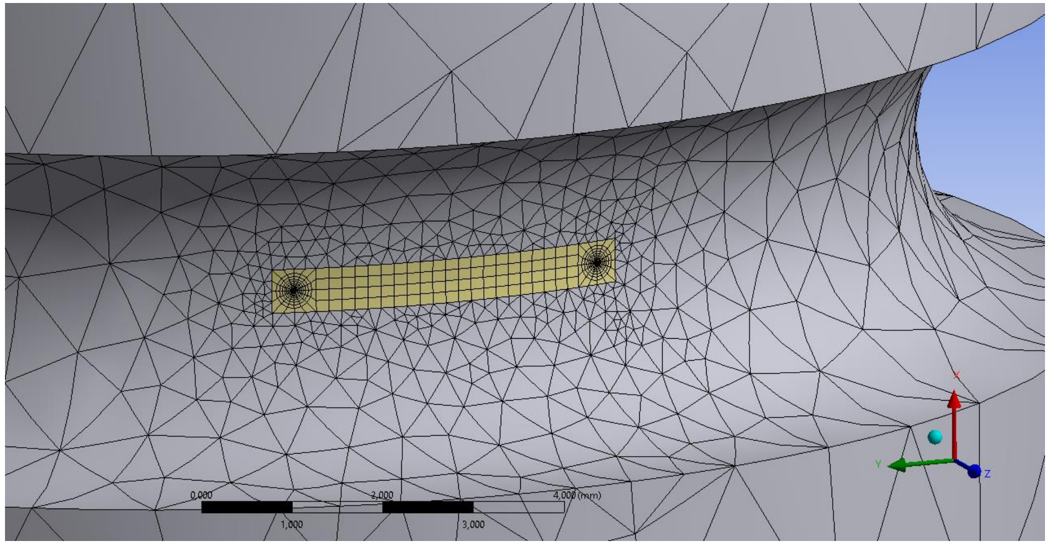
Parametric Analysis

- In order to push the morphing action to the **limit**, which depends on mesh **quality** after morphing, a parametric analysis was performed.
- Mesh morphing allowed to impose **large displacement** to the mesh preserving **numerical stability**.
- Results reported cover a quite wide **range of dimensions and aspect ratios** which led to consider the suggested method **applicable** in this kind of problems.

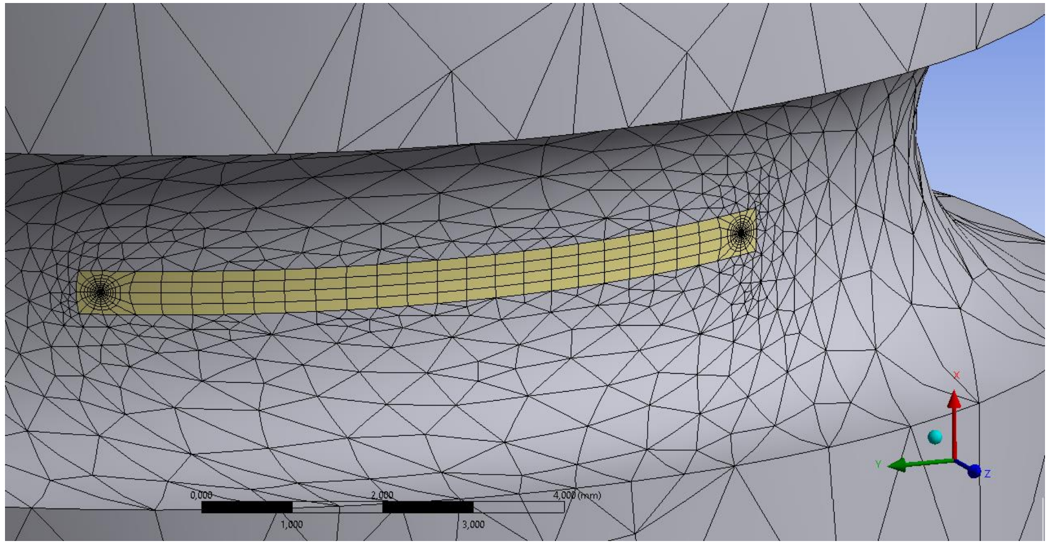
a [mm]	b [mm]	α [-]	SIF_{MAX} $[MPa \cdot \sqrt{mm}]$	SIF_{min} $[MPa \cdot \sqrt{mm}]$
1.30	1.30	1.00	400.24	227.95
1.30	1.95	0.66	340.16	262.24
1.30	3.90	0.33	340.93	203.33
1.30	13.00	0.10	356.86	144.48
1.40	1.40	1.00	396.69	237.33
1.40	2.10	0.66	348.92	271.29
1.40	4.20	0.33	350.55	211.02
1.40	14.00	0.10	365.17	156.13
1.60	1.60	1.00	433.75	238.47
1.60	2.40	0.66	368.85	275.16
1.60	4.81	0.33	368.49	238.08
1.60	16.00	0.10	381.13	178.96
1.70	1.70	1.00	394.92	263.64
1.70	2.55	0.66	375.78	296.97
1.70	5.11	0.33	379.40	235.43
1.70	17.00	0.10	390.08	188.27
1.80	1.80	1.00	452.18	245.86
1.80	2.70	0.66	389.13	284.09
1.80	5.40	0.33	386.92	261.38
1.80	18.00	0.10	398.12	201.58

Parametric Analysis

- An example of the resulting morphed mesh is reported. It is possible to notice the displacement imposed to obtain the final configuration.



baseline crack



morphed crack

Crack Growth Simulation

- Finally a test on the **crack growth** simulation feasibility was performed adopting the Paris-Erdogan law:

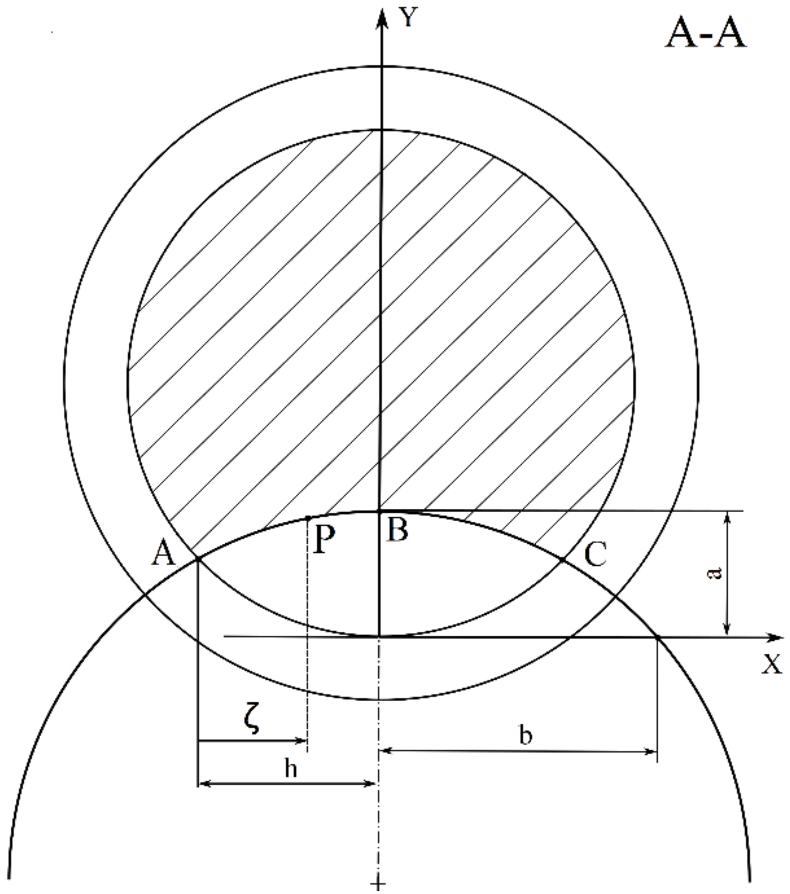
$$\frac{da}{dN} = C\left(\Delta K_{eff}\right)^{m}$$

in which **C** and **m** coefficients were extracted from Appendix 16 of RCC-MR standard.

- Since **maximum** value of KI is attained **near the surface** point of the crack and the **minimum** value at the **deepest** point, **a first two-parameter model** was adopted to approximate the crack profile.

Crack Growth Simulation

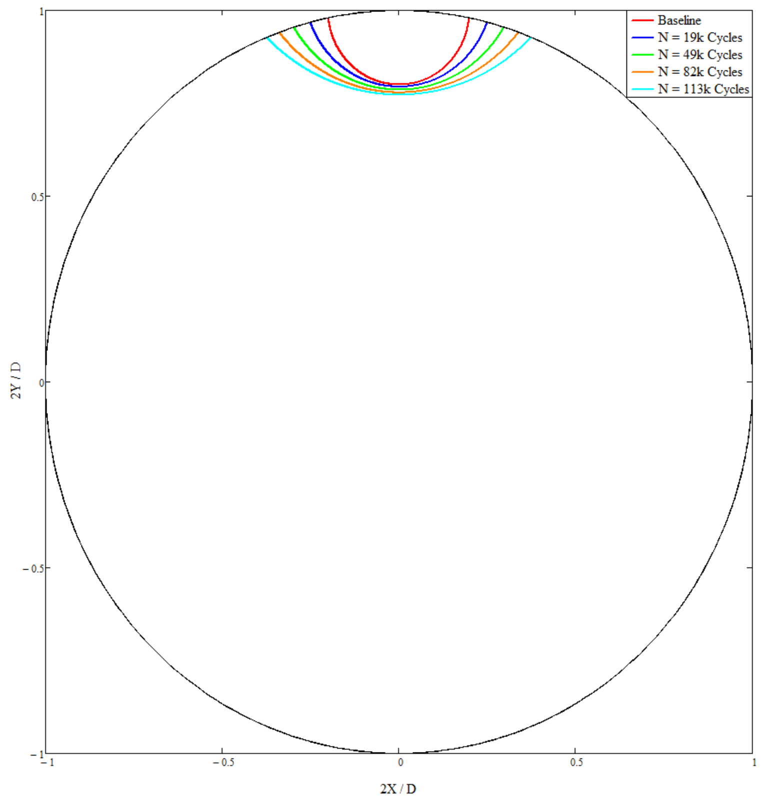
- The profile is approximate by a circular arc
- The center can move along the symmetry axis (**Y** axis) to represent different aspect ratios crack profiles.
- The whole crack profile is defined by three points: two on the perimeter of the reduced cross section (**A** and **C**) and one on the symmetry axis (point **B**).



Crack Growth Simulation

- **FEM** analysis is used to obtain the SIFs distribution
- The **Paris-Erdogan** law was adopted to determine the position of the three points describing the crack front.
- A **1.6x1.6 mm** crack after **113k cycles** of a zero-to-maximum load cycle assumes **1.82x3.48mm** dimension

a [mm]	b [mm]	α []	N_{cyc} [kCycles]	SIF_{MAX} $[MPa \cdot \sqrt{mm}]$	SIF_{min} $[MPa \cdot \sqrt{mm}]$
1.60	1.60	1.00	0	433.75	238.47
1.65	2.15	0.76	18.83	385.89	265.31
1.71	2.56	0.63	48.90	377.55	288.93
1.77	3.27	0.54	81.71	381.82	310.61
1.82	3.48	0.47	113.08	-	-



crack front advancement at 113k cycles

Conclusions

- In this work an assessment of **mesh morphing** techniques applied to the fracture mechanics was presented, adopting RBF Morph™ and ANSYS® Workbench™.
- Firstly the mesh morphing approach reliability was investigated comparing the results obtained using **morphed meshes** with cracks obtained using the **ANSYS Workbench Fracture Tool**.
- The mesh morphing approach was then tested in a **parametric analysis** in which both crack dimensions and aspect ratios were varied.
- Then the use of mesh morphing was applied to the **simulation of crack growth**: SIFs values were retrieved along the crack length by means of FEM analyses and the Paris-Erdogan law was employed to determine the crack advancement.
- The crack shape is updated after the FEM analysis applying the mesh morphing approach presented. The procedure can be iterated to perform further steps of crack growth simulations.

Further improvements

- A more complex model for the crack growth is being implemented in the workflow: each node of the crack can be moved according to the local SIF value.
- Implementation of the methodology to investigate 3D crack fronts

THANK YOU!

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