Shape optimization using structural adjoint and RBF mesh morphing

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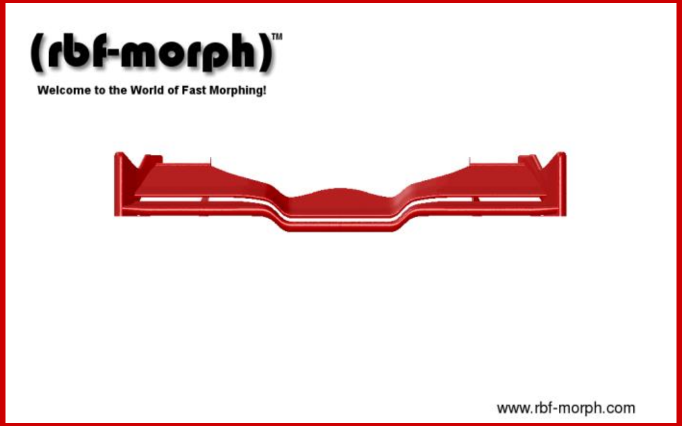

Introduction

- # Implemented in RBF4AERO

  'Innovative benchmark technology for aircraft engineering design and efficient design phase optimisation' partially funded by the EUs 7th Framework Programme (FP7-AAT, 2007-2013) under Grant Agreement no. 605396.

- # Adjoint preview and **adjoint sculpting**

  two optimization workflows using adjoint-based sensitivity data are proposed
  - Preview: a number of shape parameters are evaluated using sensitivities.
  - **Sculpting: the adjoint solver directly suggests shape evolution by imposing each boundary node movement**

- # RBF at the core of the method

  Radial Basis Functions (RBF) used to link the numerical analysis and optimization, exploited also for advanced tasks.

- # 10 years of experience in RBF + adjoint

  RBF Morph Fluent Add On development started in 2007 and was coupled to the very first versions of the adjoint CFD solver of ANSYS Fluent. RBF Morph Stand Alone suitable for Open Foam was developed in 2012. RBF Morph ACT extension for ANSYS Mechanical was released in 2015.

Motivation

- Human interaction as a bottleneck

  The technological evolution allows now to solve complex problems in short time, making human interaction more relevant and sometime a bottleneck. Automatic workflows are attractive

- Optimization is often carried manually

- Optimal shape parameterization is difficult

  Analizable systems are always more complex, making physical phenomena difficult to be predicted. An optimal shape parameterization is hard to be chosen especially in a multi-physics scenario

- Advanced manufacturing processes

  Evolution of manufacturing processes (i.e AM) requires advanced free-form optimization workflows.
  Current practice is focused on standard production technology

Motivation



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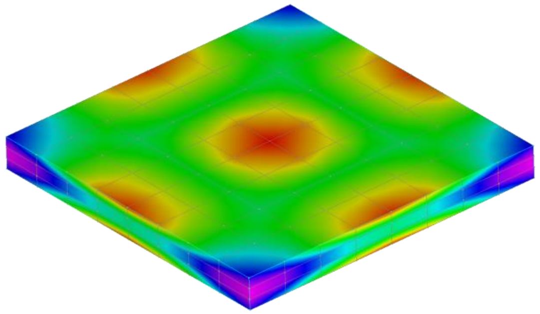
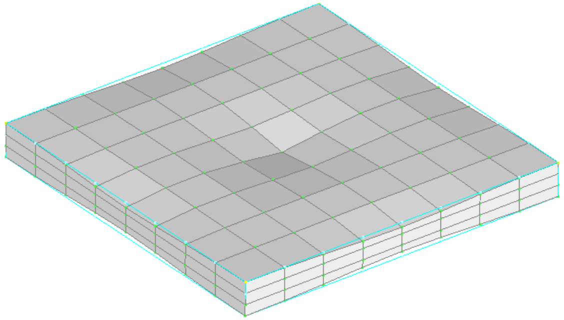
Sensitivity analysis

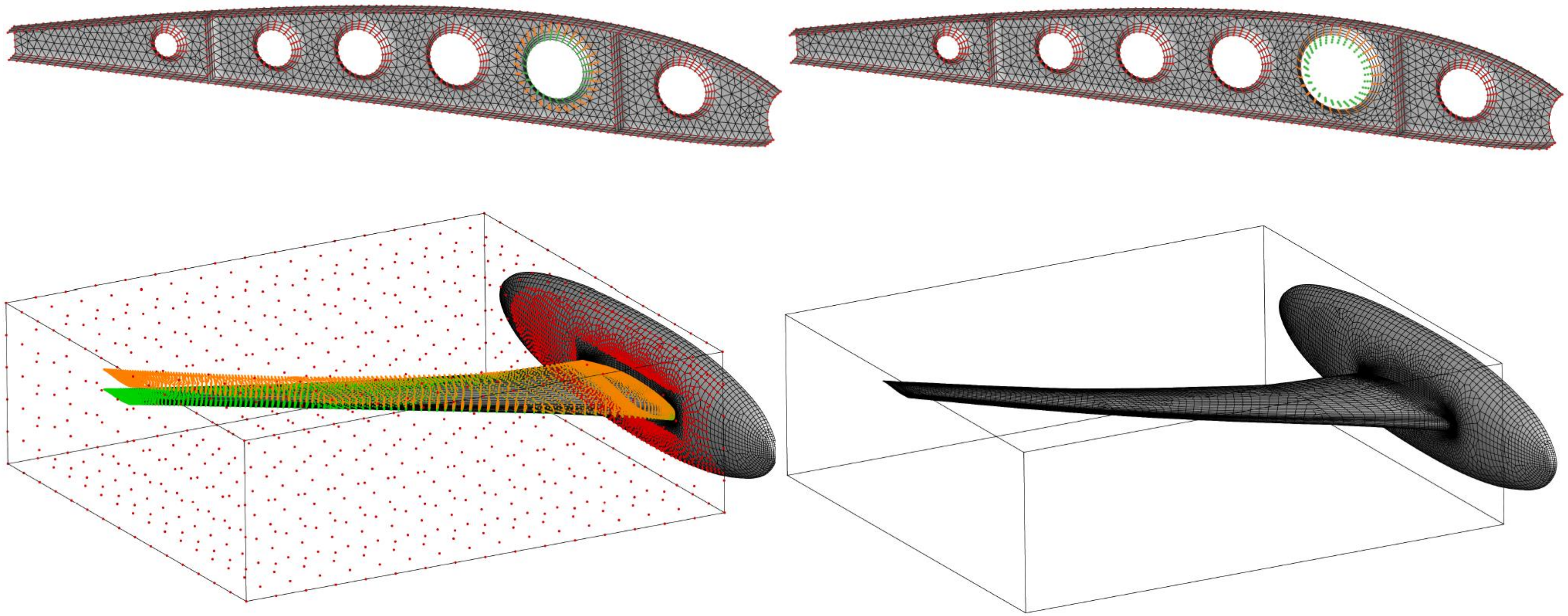
- System behavior depends on BC
  The result of the calculation changes depending on how sensitive is the system to BC variations.
- Several methods to achieve sensitivities
  Finite differencing, complex-step differentiation, automatic differentiation, adjoint method
- CFD and in-house **FEM adjoint solver**
  ANSYS® Fluent®, NTUA OpenFOAM® implementation, **Adjoint variable continuum-discrete in-house solver**





Shape parameterization

- Radial Basis Functions



Shape parameterization

- Radial Basis Functions

$$f(\mathbf{x}) = \sum_{i=1}^{m} \gamma_i \phi(\|\mathbf{c}_i - \mathbf{x}\|) + p(\mathbf{x}) \qquad p(\mathbf{x}) = \beta_1 + \beta_2 x_1 + \beta_3 x_2 + \ldots + \beta_{n+1} x_n$$

$$f(\mathbf{c}_i) = g_i$$

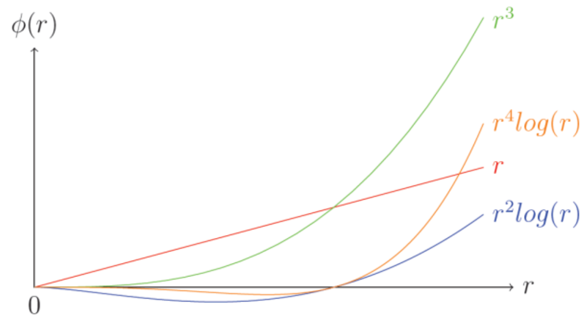
$$\begin{cases} f^x(\mathbf{x}) = \sum_{i=1}^{m} \gamma_i^x \phi(\|\mathbf{c}_i - \mathbf{x}\|) + \beta_1^x + \beta_2^x x_1 + \beta_3^x x_2 + \beta_4^x x_3 \\ f^y(\mathbf{x}) = \sum_{i=1}^{m} \gamma_i^y \phi(\|\mathbf{c}_i - \mathbf{x}\|) + \beta_1^y + \beta_2^y x_1 + \beta_3^y x_2 + \beta_4^y x_3 \\ f^z(\mathbf{x}) = \sum_{i=1}^{m} \gamma_i^z \phi(\|\mathbf{c}_i - \mathbf{x}\|) + \beta_1^z + \beta_2^z x_1 + \beta_3^z x_2 + \beta_4^z x_3 \end{cases} \qquad \mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} \qquad \mathbf{c} = \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix}$$

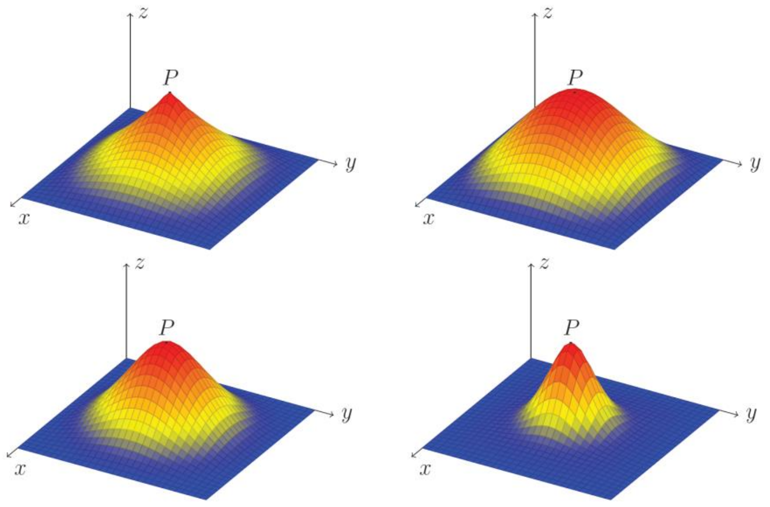
$$\begin{bmatrix} \mathrm{M} & \mathrm{P} \\ \mathbf{P}^T & 0 \end{bmatrix} \begin{Bmatrix} \gamma \\ \beta \end{Bmatrix} = \begin{Bmatrix} \mathrm{g} \\ 0 \end{Bmatrix} \qquad M_{ij} = \phi(\|\mathbf{c}_i - \mathbf{c}_j\|) \qquad P_j = [1 \quad x_1 \quad x_2 \quad \ldots \quad x_n]$$

- Radial Basis Functions

Name	Abbreviation	$\phi(r)$
Polyharmonic spline	PHS	r^n, n odd
		$r^n log(r)$, n even
Thin plate spline	TPS	$r^2 log(r)$
Multiquadric biharmonics	MQB	$\sqrt{a^2 + (\epsilon r)^2}$
Inverse multiquadric biharmonics	IMQB	$\frac{1}{\sqrt{a^2+(\epsilon r)^2}}$
Quadric biharmonics	QB	$1 + (\epsilon r)^2$
Inverse quadric biharmonics	IQB	$\frac{1}{1+(\epsilon r)^2}$
Gaussian biharmonics	GS	$e^{-\epsilon r^2}$

Name	Abbreviation	$\phi(\zeta)$
Wendland C^0	$C0$	$(1-\epsilon\zeta)^2$
Wendland C^2	$C2$	$(1-\epsilon\zeta)^4(4\epsilon\zeta+1)$
Wendland C^4	C4	$(1-\epsilon\zeta)^6(\frac{35}{3}\epsilon\zeta^2+6\epsilon\zeta+1)$



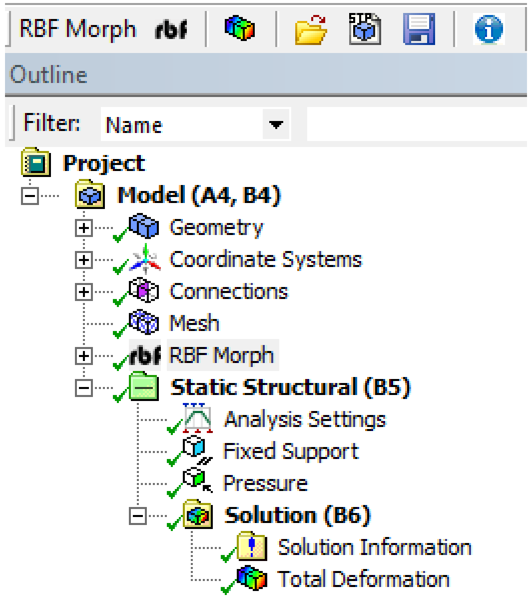


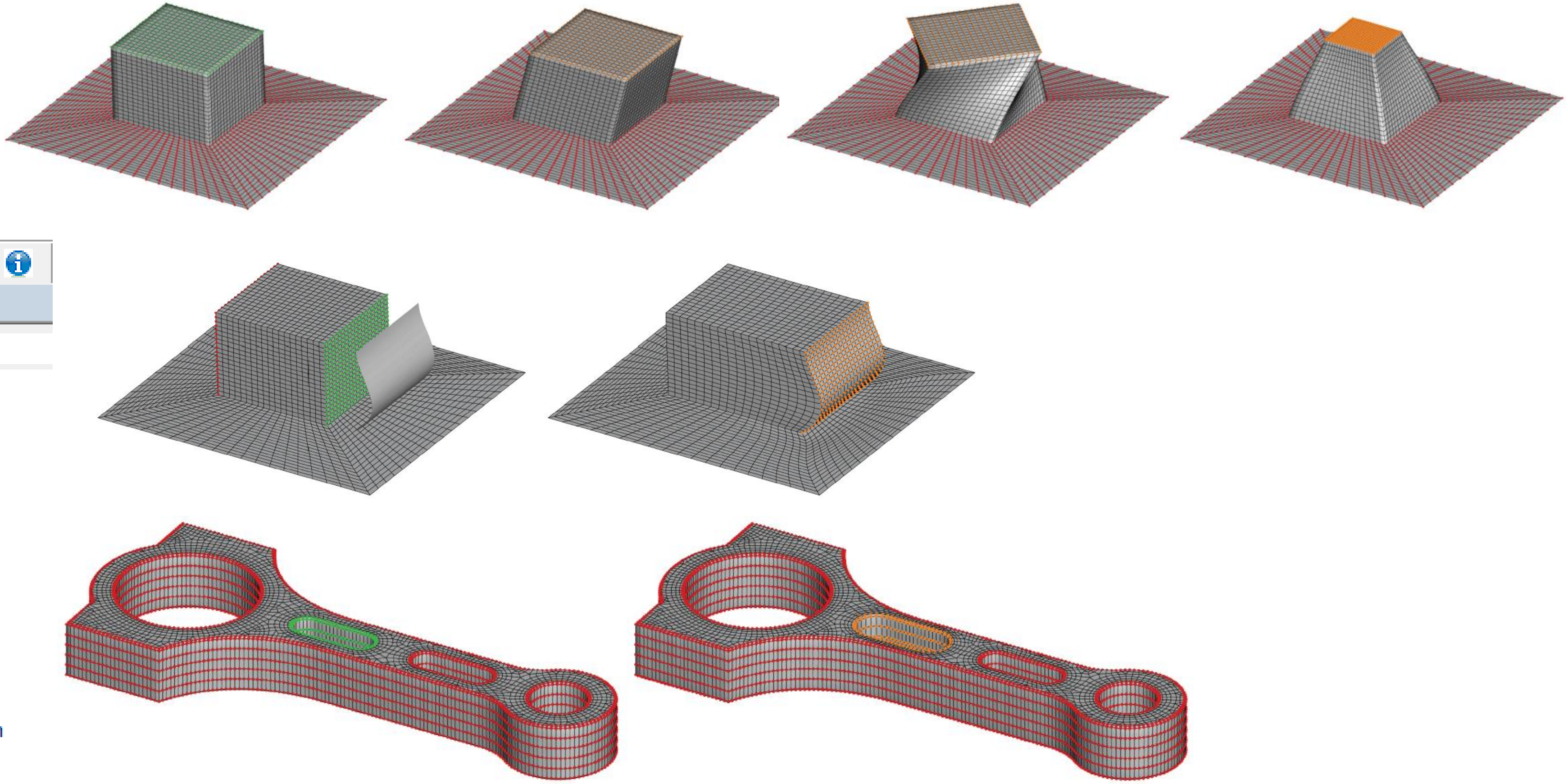
Shape parameterization

- Radial Basis Functions: basic shape modifications

ANSYS®

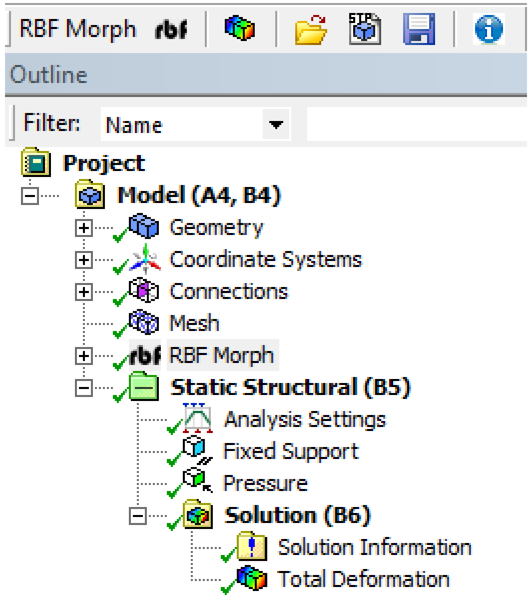
(rbf-morph)™

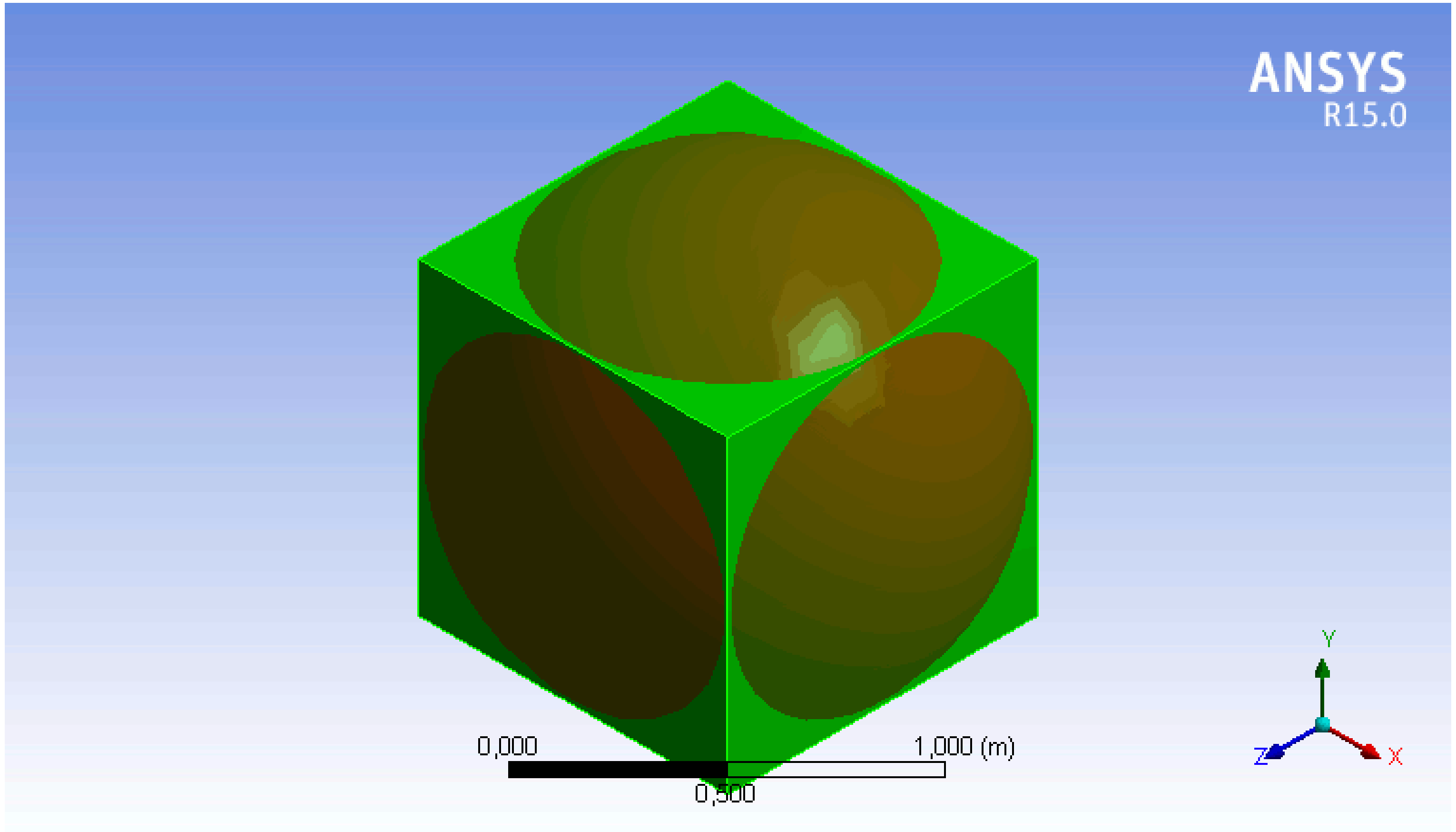




Shape parameterization

- Squaring of the circle? Sphering of the Cube!





Adjoint sculpting

- How to provide meaningful shape variations?
  Sensitivity data naturally sculpts surfaces
- Gradient-based algorithm
  Direction search and step given by adjoint solution
- Noisy sensitivity data
  Especially for CFD applications, noisy sensitivity maps can result in ill-posed problems
- Design constraints
  Packaging and functional constraints can be maintained using RBF

Adjoint sculpting: noise filtering

- Noise filtering: RBF least squares smoothing

  Data is sub-sampled, error between full and reduced problems is minimized

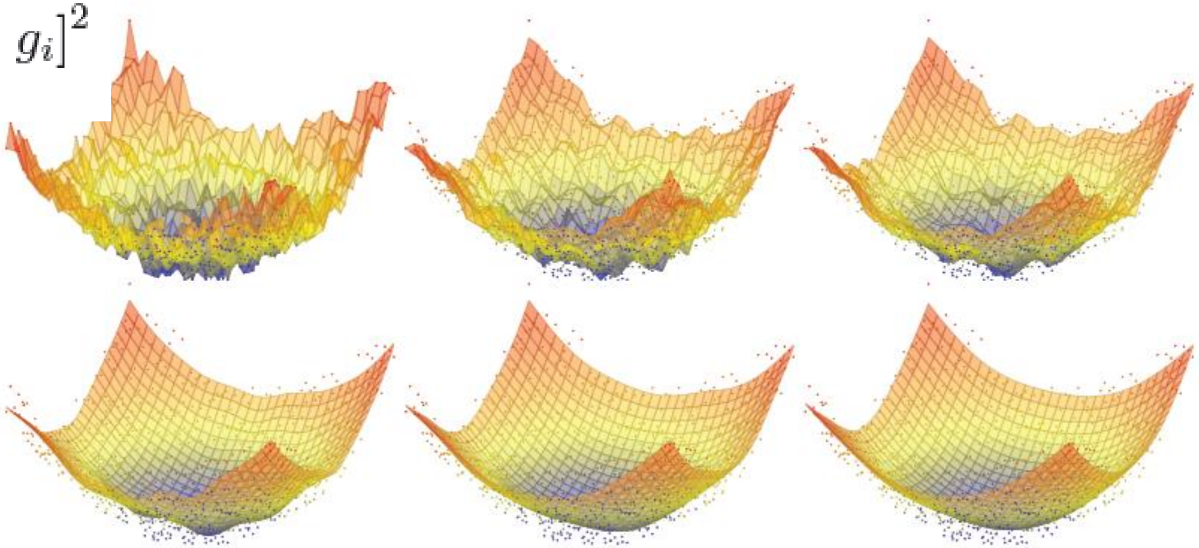
$$f_c(\mathbf{x}) = \sum_{j=1}^{m_c} \gamma_j \phi(\|c_{cj} - \mathbf{x}\|) \qquad \sum_{i=1}^{m} [f_c(c_i) - g_i]^2$$

$$\begin{bmatrix} \mathrm{M} & \mathrm{P} \\ \mathrm{P}_c^T & 0 \end{bmatrix}^T \begin{bmatrix} \mathrm{M} & \mathrm{P} \\ \mathrm{P}_c^T & 0 \end{bmatrix} \begin{Bmatrix} \gamma \\ \beta \end{Bmatrix} = \begin{bmatrix} \mathrm{M} & \mathrm{P} \\ \mathrm{P}_c^T & 0 \end{bmatrix}^T \begin{Bmatrix} \mathrm{g} \\ 0 \end{Bmatrix}$$

$$M_{ij} = \phi(\|c_i - c_{cj}\|)$$

$$\mathrm{Ax} = \mathrm{b}$$

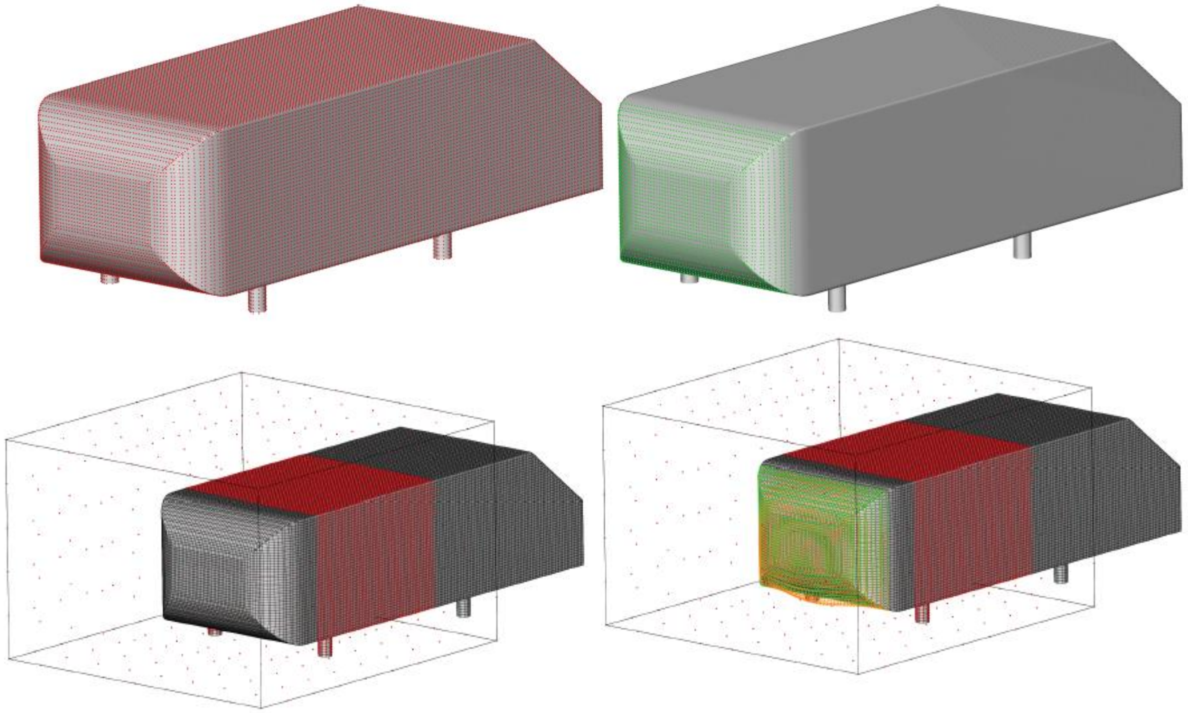
$$(\mathrm{A}^T \cdot \mathrm{A}) \cdot \mathrm{x} = (\mathrm{A}^T \cdot \mathrm{b})$$

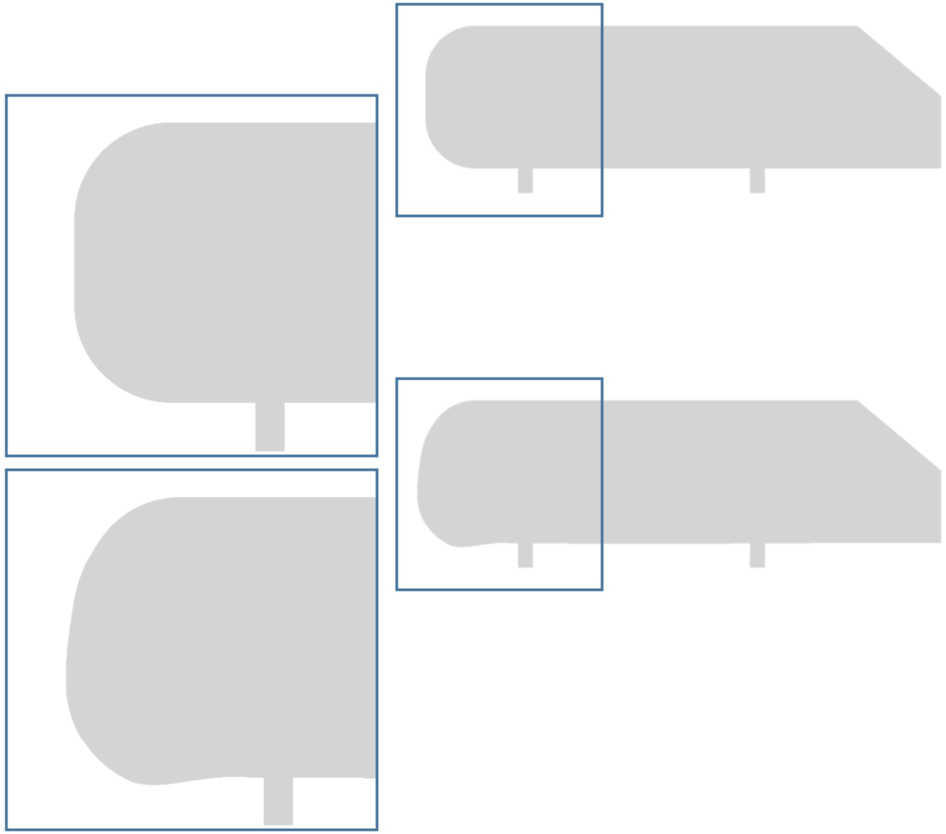


Adjoint sculpting: RBF set-up

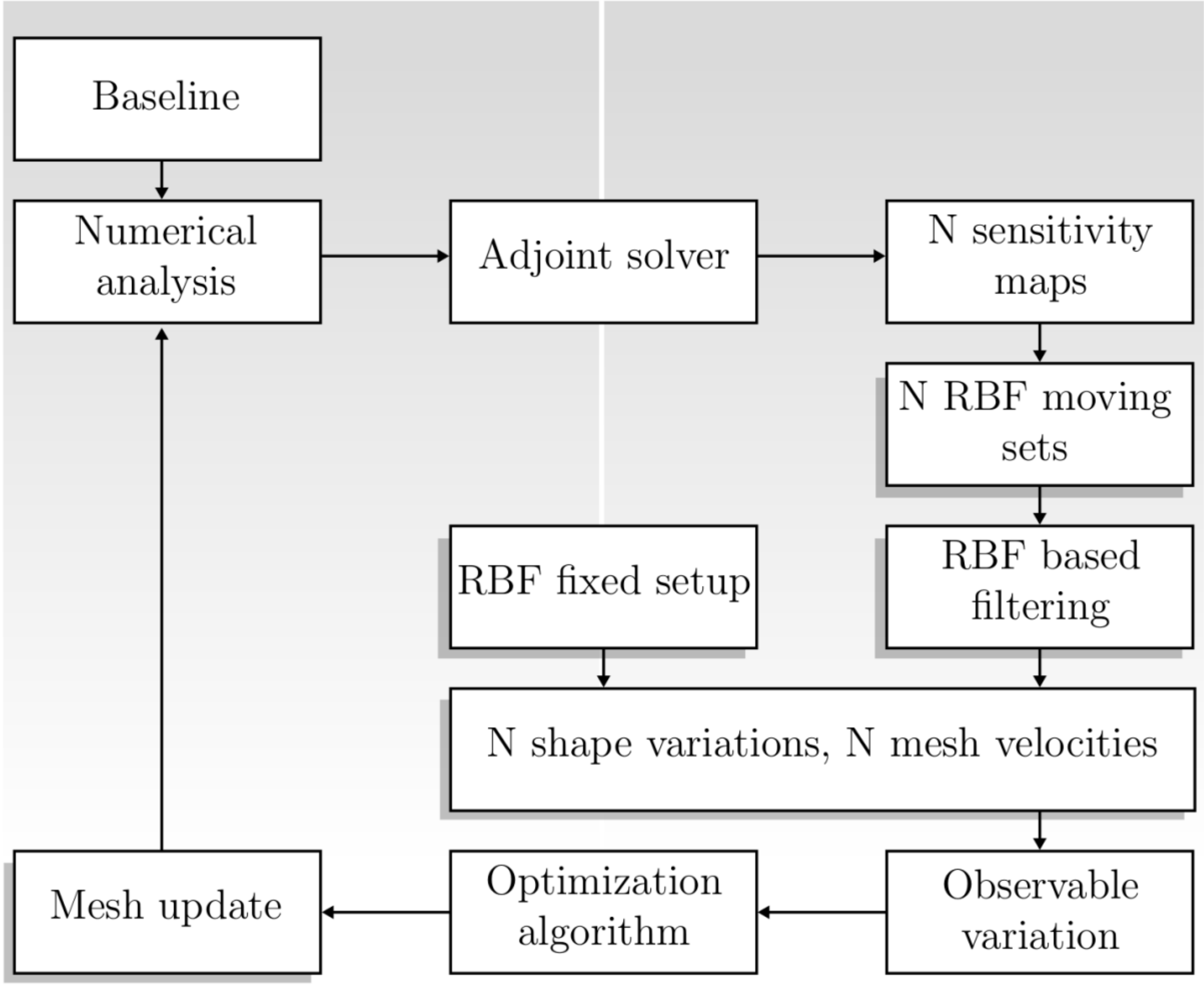
- Evolutionary reshape: new set-up at each iteration

  Fixed set-up to force constraints, moving set-up to apply evolutionary shape variations





Adjoint sculpting: workflow



Adjoint sculpting: in-house FEM adjoint solver

- Implemented in **ANSYS Mechanical** using ACT

  Adjoint variable continuum-discrete in-house solver. Works for linear 3-d hexa elements. Two linear static analyses are solved for the same model (original problem and augmented problem). Observable can be the displacement of a FEM node. Node position sensitivities are computed on all the nodes of the surface.

- Mesh morphing by **RBF Morph ACT extensions**

  Computed sensitivities computed on the solid surface are optionally filtered and then transformed in an input map for the RBF Morph ACT extension.

- Adjoint sculpting loops

  At the moment the calculation of the base and augmented solution is manually triggered by the user. Morphed mesh can be maintained for a subsequent step or reverted to the previous one if constrains are met or the mesh quality limit is reached.

- Back2CAD

  A NURBS representation of the optimised surfaces can be achieved by CAD morphing (RBF Morph) or by a CAD reconstruction (FEModeler) that preserves the topology of the base underlying geometrical model.

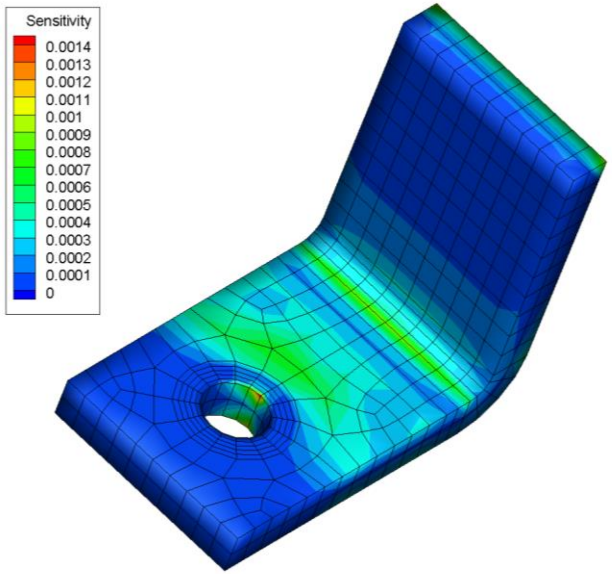
Adjoint sculpting: in-house FEM adjoint solver

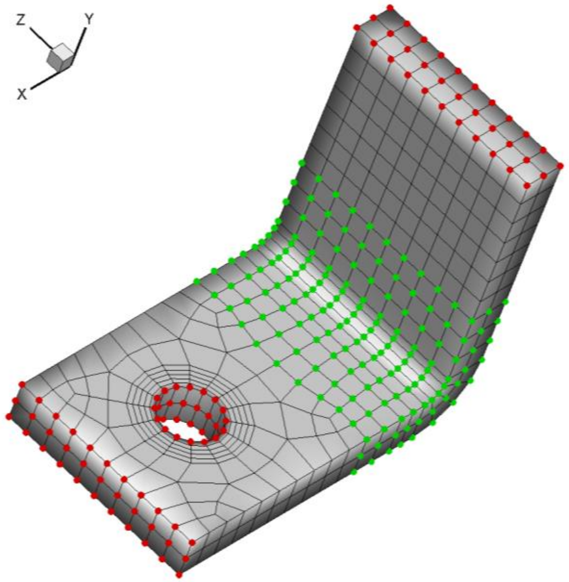
- The discrete case
  - Observed function $\psi = f(\mathbf{X}(u), u)$ (for instance the displacement of a point) total derivative vs. the input parameter u (for instance the variation of a grid node coordinate)
  - $\frac{d\psi}{du} = \frac{\partial\psi}{\partial u} + \frac{\partial\psi}{\partial \mathbf{X}}\frac{\partial \mathbf{X}}{\partial u}$
  - A linear system $\mathbf{K}\,\mathbf{X} = \mathbf{F}$ can be differentiated
  - $\mathbf{K}\frac{\partial \mathbf{X}}{\partial u} + \mathbf{X}\,\frac{\partial \mathbf{K}}{\partial u} = \frac{\partial \mathbf{F}}{\partial u} \qquad \mathbf{K}\frac{\partial \mathbf{X}}{\partial u} = \frac{\partial \mathbf{F}}{\partial u} - \mathbf{X}\,\frac{\partial \mathbf{K}}{\partial u}$
  - And the sensitivity computed using the direct method (fast for many observables and a few input parameters)
  - $\frac{d\psi}{du} = \frac{\partial\psi}{\partial u} + \frac{\partial\psi}{\partial \mathbf{X}}\mathbf{K}^{-1}\left(\frac{\partial \mathbf{F}}{\partial u} - \mathbf{X}\,\frac{\partial \mathbf{K}}{\partial u}\right)$
  - The adjoint system can be solved once (it’s a function of observable regardless the input varied)
  - $\mathbf{K}\lambda = \frac{\partial\psi}{\partial \mathbf{X}}^{\mathrm{T}}$
  - And the sensitivity computed using the adjoint method (fast for a few observables and many input parameters)
  - $\frac{d\psi}{du} = \frac{\partial\psi}{\partial u} + \lambda^{\mathrm{T}}\left(\frac{\partial \mathbf{F}}{\partial u} - \mathbf{X}\,\frac{\partial \mathbf{K}}{\partial u}\right)$

Bracket

- # Displacement minimization

  Young's modulus = 200 Gpa, Poisson's ratio = 0.3, F = 5000 N along the x axis, fixed hole
  Step length is varied at each cycle to assure a maximum displacement of 1 mm





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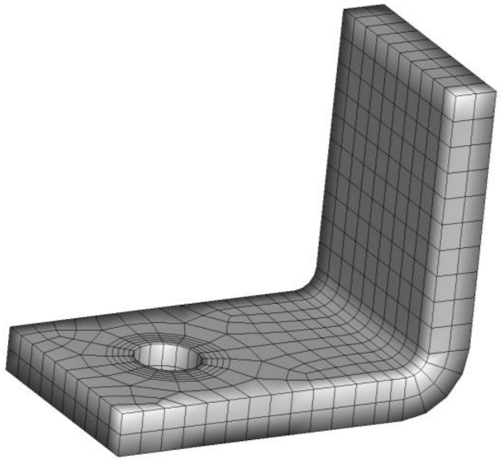
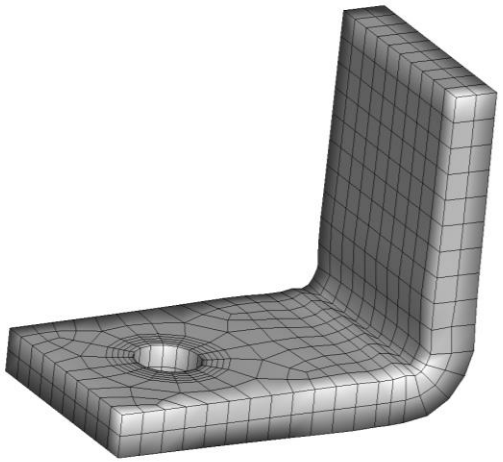
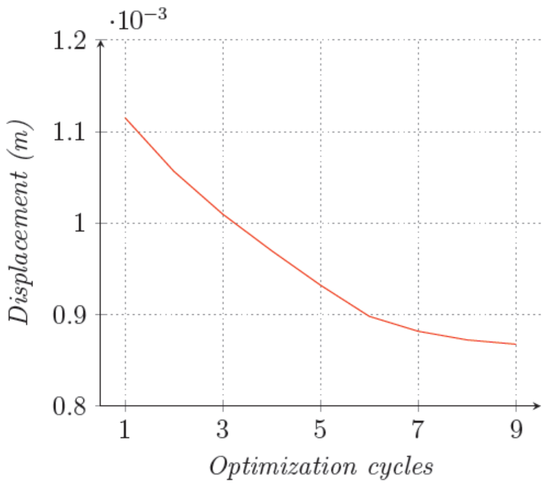
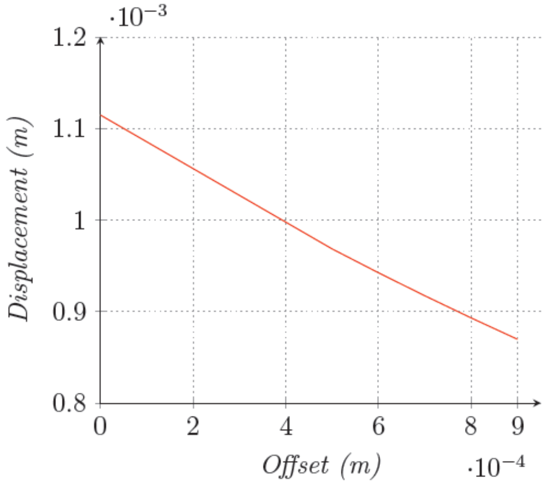








- **Displacement reduction**

  22% after 9 cycles with regard to original displacement

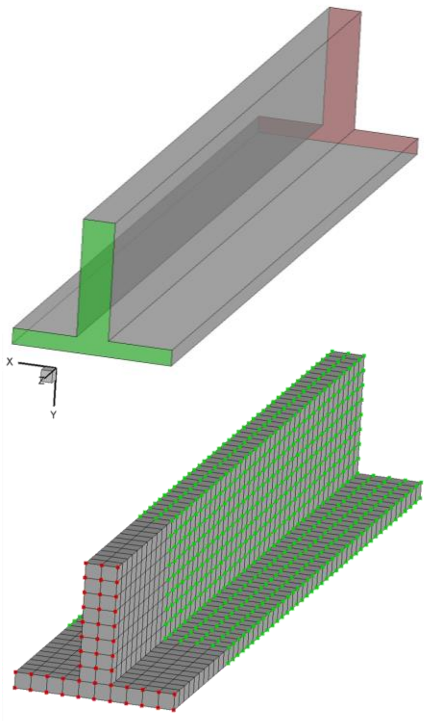
- **Optimal shape variation**

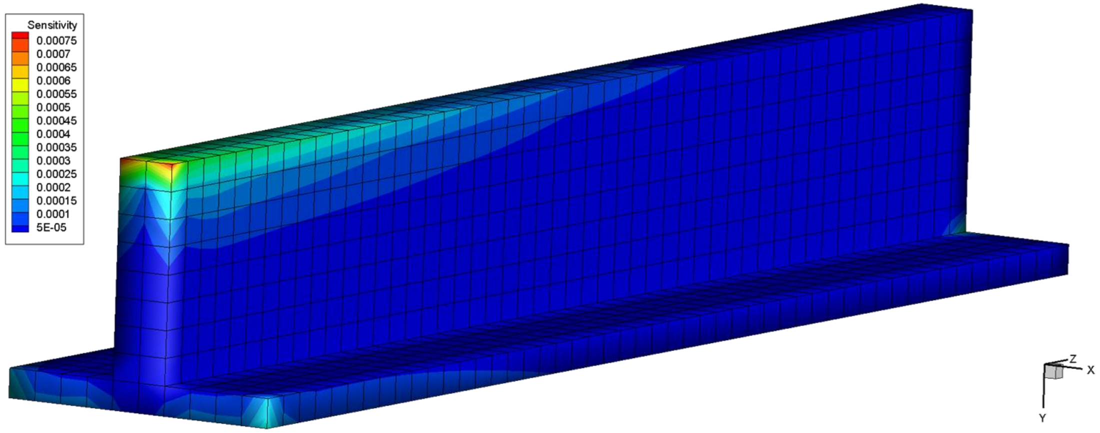
  6% increase in mass compared to 9% increase achieved employing zero order methods with constant thickness

T-beam

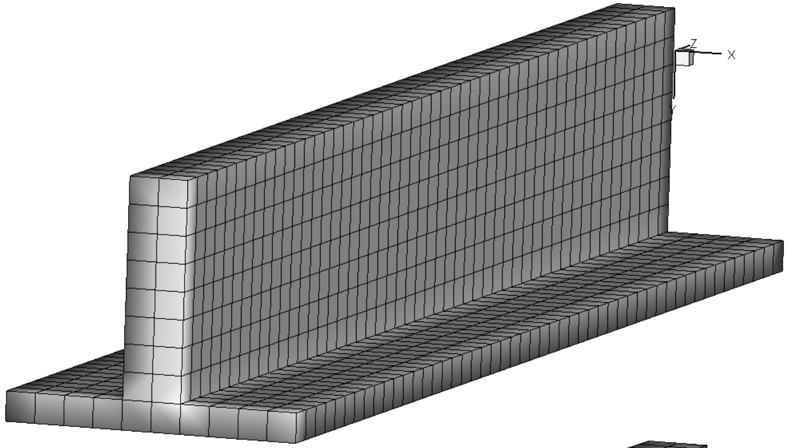
- # Displacement free end minimization

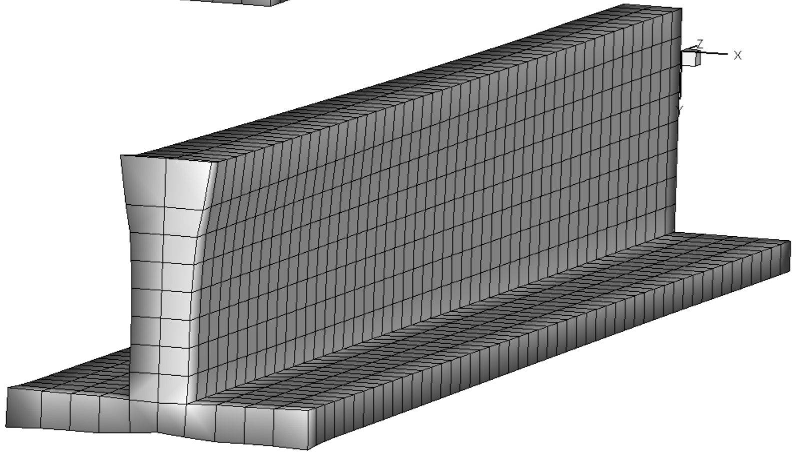
  Young's modulus = 200 GPa, Poisson's ratio = 0.3, F = 10000 N load

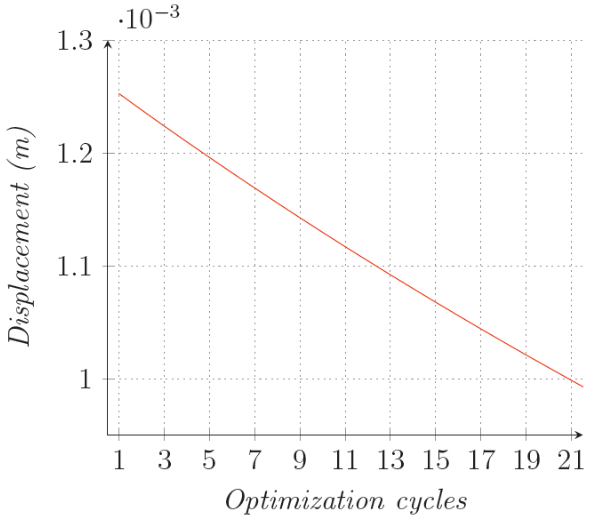




T-beam







- **Achieved results**

  25% reduction after 21 cycles with regard to original displacement

Conclusions

- ## Adjoint-sculpting automatic workflow

  The adjoint shape optimization workflow was shown for FEM applications but can be applied also for other physics. CFD implementation using **Open Foam** + RBF Morph Stand Alone and **ANSYS Fluent** + RBF Morph Fluent Add On are well established.

- ## RBF for shape parameterization

  Automatic reshape in adjoint sculpting can be enabled thanks to advanced RBF mesh morphing. Packaging and functional constraints are maintained. A new CAD representation can be generated.

- ## RBF for advanced tasks

  RBF are used to filter noisy data, to define implicit surfaces for offset or projection modifiers.

- ## FEM and CFD applications

  Tools and workflows implemented in RBF4AERO were tested in FEM and CFD applications. A rich collection of aeronautical applications is available on www.rbf4aero.eu. Industrial applications of RBF mesh morphing for ANSYS solvers (ANSYS Mechanical and ANSYS Fluent) are available on www.rbf-morph.com and on www.ansys.com. The complete description of the adjoint solver is available in the PhD Thesis of Dr. C. Groth “Adjoint-based shape optimization workflows using RBF”. https://www.researchgate.net/publication/316788911_Adjoint-based_shape_optimization_workflows_using_RBF

RBF4AERO

THANKS FOR YOUR ATTENTION

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