Automatic Optimization Method Based on Mesh Morphing Surface Sculpting Driven by Biological Growth Method: an Application to Coiled Spring Section Shape

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ICCS 2021 – International Conference on Computational Science – 16-18 June, 2021

Outline

- Introduction
- Radial Basis Functions Mesh Morphing
- Biological Growth Method (BGM) Background
- Parameter-less Optimization
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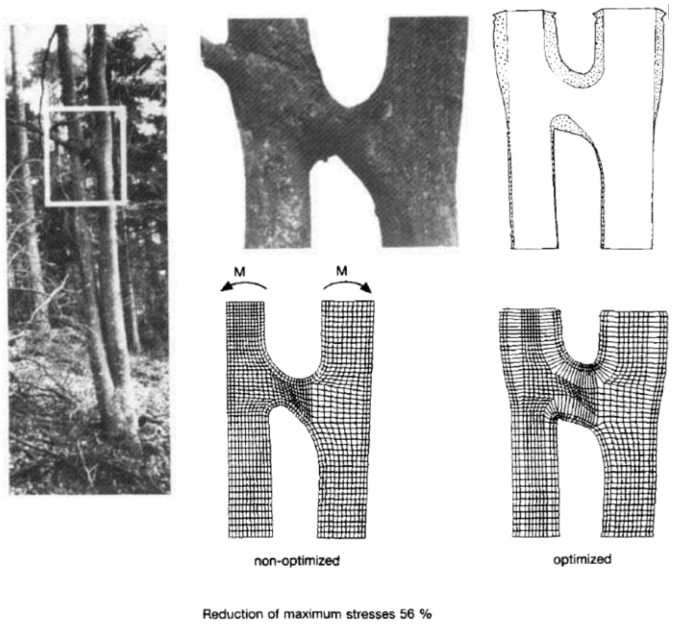
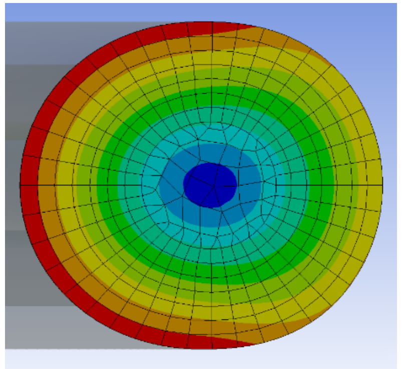
- Conclusions

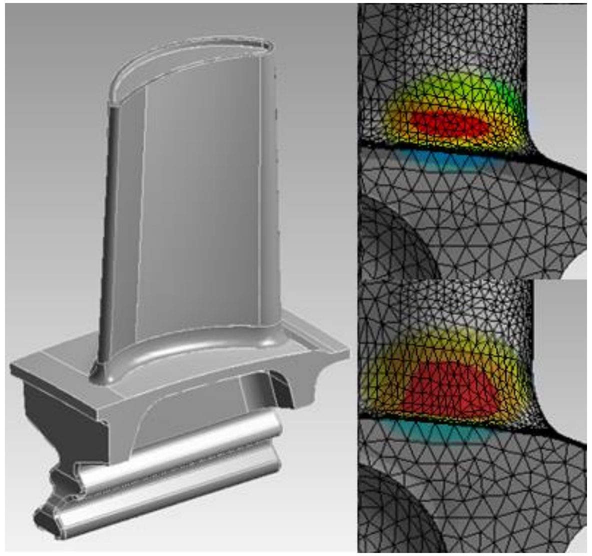


$$s(\boldsymbol{x}) = \sum_{i=1}^{N} \gamma_i \varphi\left(\left\|\boldsymbol{x} - \boldsymbol{x}_{k_i}\right\|\right) + h(\boldsymbol{x})$$

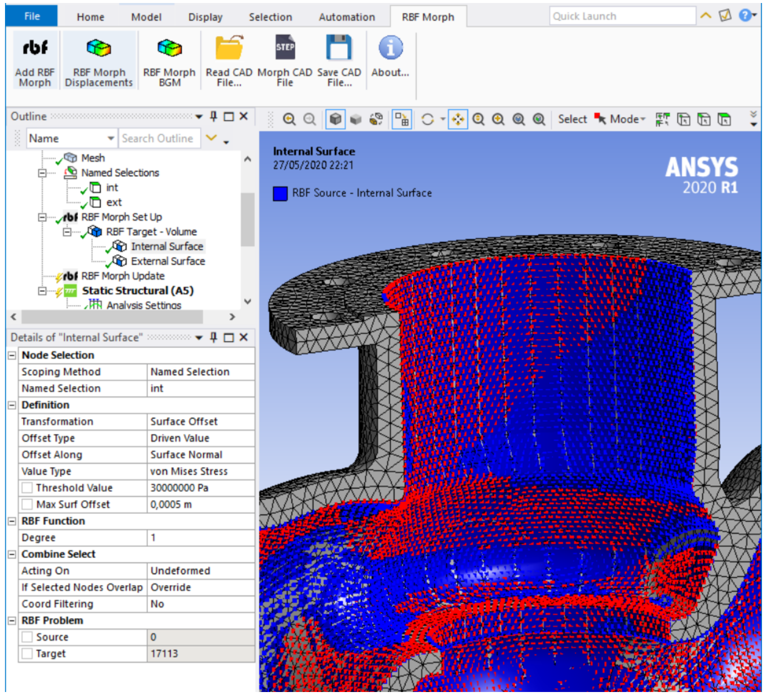


Introduction and motivation

- Nowadays each design process requires **Optimization**, thus **optimization techniques** are gaining a high importance in design and manufacturing of new products
- In the product design, numerical simulations, as Finite Element Method (FEM), are employed to **virtually test different configurations**
- Nevertheless, research of an **optimal configuration** can be time-consuming and techniques to automate both model generation and configuration optimization are requested
- **Mesh morphing** is an innovative technique that allow to reduce time needed to obtain a new configuration of a numerical model by applying shape modification directly to the computational grid



Introduction and motivation



- **Mesh Morphing** can be driven using several approaches, one of the most promising is the Biological Growth Method (BGM)
- **BGM is inspired** by the way in which **natural tissues** react to a surface load, let the tissues to growth in order to reduce surface stresses
- BGM and Mesh Morphing can be **successfully coupled** to obtain a surface sculpting methodology which is effective in mechanical component optimization
- Methodology has been developed and is presented in the framework of **ANSYS Mechanical** Finite Element Analysis (FEA) tool using the **RBF Morph ACT** extension as mesh morpher

RBF Mesh Morphing

- Radial Basis Functions (RBF) are a mathematical tool capable to **interpolate** in a generic point of the space a function **known** in a discrete set of points (**source points**)
- The interpolating function is composed by a **radial basis** and by a **polynomial**:

$$s(\boldsymbol{x}) = \underbrace{\sum_{i=1}^{N} \gamma_i \varphi\left(\left\| \boldsymbol{x} - \boldsymbol{x}_{k_i} \right\| \right)}_{\text{radial basis}} + \underbrace{h(\boldsymbol{x})}_{\text{polynomial}}$$

radial basis

polynomial

distance from the i-th source point

RBF Mesh Morphing

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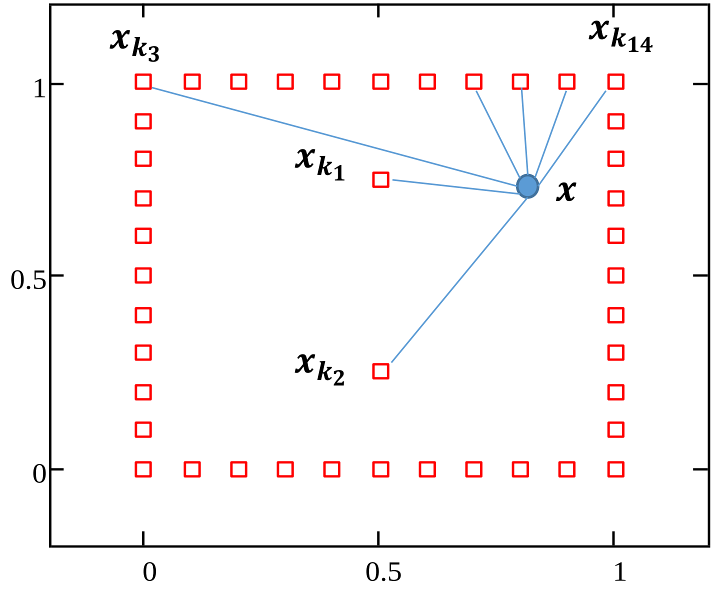
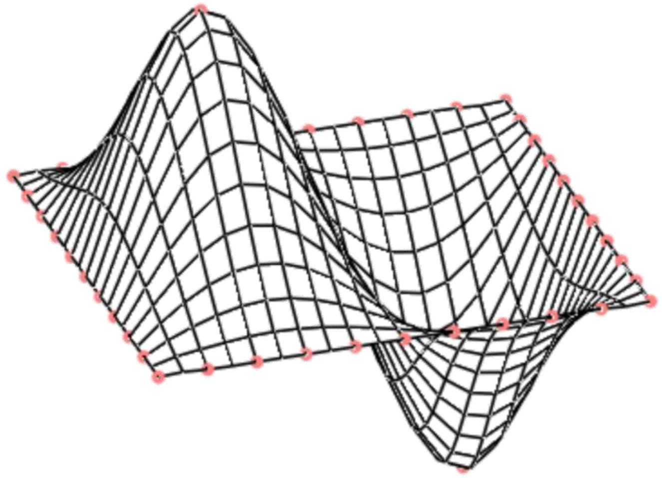




RBF Mesh Morphing

- If evaluated on the **source points**, the interpolating function gives exactly the input values

$$s(\boldsymbol{x}_{k_i}) = g_i \qquad h(\boldsymbol{x}_{k_i}) = 0 \qquad 1 \le i \le N$$

- The RBF problem (evaluation of coefficients γ and β) is associated to the solution of **the linear system**, in which **M** is the interpolation matrix, **P** is a constraint matrix, **g** is the vector of known values on the source points

$$\begin{bmatrix} \mathbf{M} & \mathbf{P} \\ \mathbf{P}^{\mathrm{T}} & 0 \end{bmatrix} \begin{pmatrix} \boldsymbol{\gamma} \\ \boldsymbol{\beta} \end{pmatrix} = \begin{pmatrix} \boldsymbol{g} \\ 0 \end{pmatrix} \qquad M_{ij} = \varphi\left(\boldsymbol{x}_{k_i} - \boldsymbol{x}_{k_j}\right) \quad 1 \le i, j \le N \qquad \mathbf{P} = \begin{bmatrix} 1 & x_{k_1} & y_{k_1} & z_{k_1} \\ 1 & x_{k_2} & y_{k_2} & z_{k_2} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{k_N} & y_{k_N} & z_{k_N} \end{bmatrix}$$

RBF Mesh Morphing

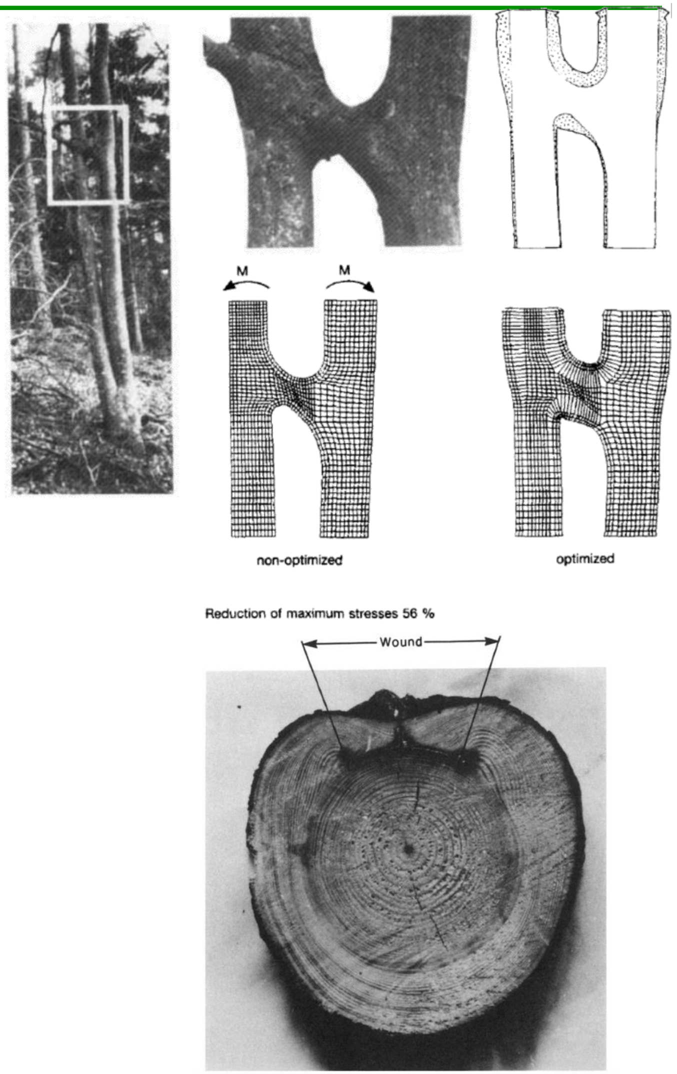
- Once solved the RBF problem each displacement component is interpolated to obtain the **displacement field**
- Several different **radial functions** (kernel) can be employed

$$\begin{cases} s_x(\mathbf{x}) = \sum_{i=1}^{N} \gamma_i^x \varphi\left(\mathbf{x} - \mathbf{x}_{\mathbf{k}_i}\right) + \beta_1^x + \beta_2^x x + \beta_3^x y + \beta_4^x z \\ s_y(\mathbf{x}) = \sum_{i=1}^{N} \gamma_i^y \varphi\left(\mathbf{x} - \mathbf{x}_{\mathbf{k}_i}\right) + \beta_1^y + \beta_2^y x + \beta_3^y y + \beta_4^y z \\ s_z(\mathbf{x}) = \sum_{i=1}^{N} \gamma_i^z \varphi\left(\mathbf{x} - \mathbf{x}_{\mathbf{k}_i}\right) + \beta_1^z + \beta_2^z x + \beta_3^z y + \beta_4^z z \end{cases}$$

RBF	φ(r)	RBF	φ(r)
Spline type (Rn)	r^n, n odd	Inverse multiquadratic (IMQ)	$\frac{1}{\sqrt{1+r^2}}$
Thin plate spline	$r^n \log(r)$ n even	Inverse quadratic (IQ)	$\frac{1}{1+r^2}$
Multiquadratic (MQ)	$\sqrt{1+r^2}$	Gaussian (GS)	e^{-r^2}

BGM Background

- **BGM** approach is based on the observation that **biological** structures growth is driven by **local** level of **stress**.
- Bones and trees' trunks are able to **adapt the shape** to mitigate the stress level due to external loads.
- The process is driven by stress **value at surfaces**. Material can be **added or removed** according to local values.
- Was proposed by Mattheck & Burkhardt in 1990*



*Mattheck C., Burkhardt S., 1990. A new method of structural shape optimization based on biological growth. Int. J. Fatigue 12(3):185-190.

BGM Background

- The BGM idea is that surface growth can be expressed as a **linear law** with respect to a given threshold value

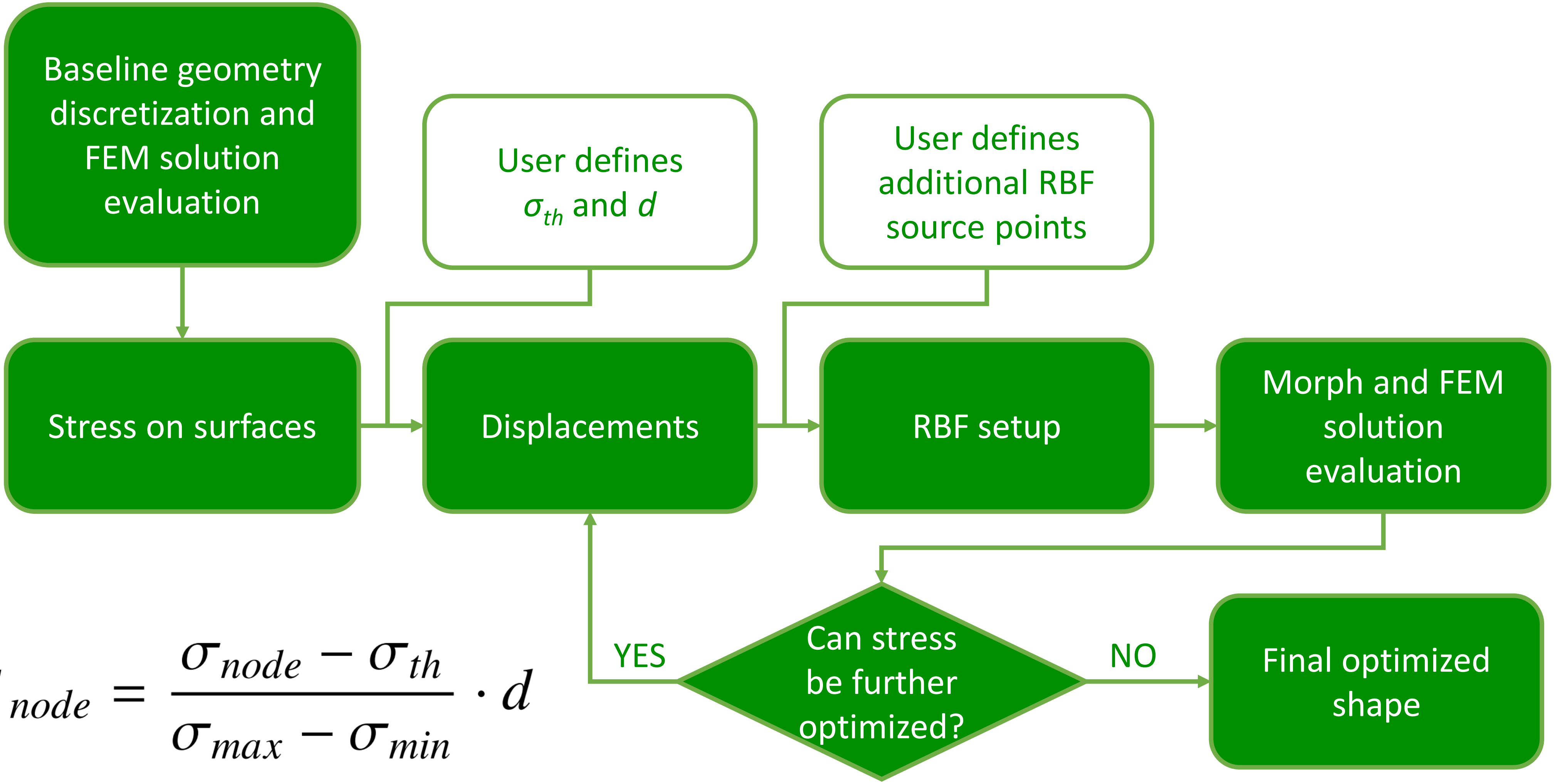
$$\dot{\varepsilon} = k\left(\sigma_{Mises} - \sigma_{ref}\right)$$

- In this study we extend the concept and different **stress types** can be used to modify the surface shape

$$S_{node} = \frac{\sigma_{node} - \sigma_{th}}{\sigma_{max} - \sigma_{min}} \cdot d$$

Stress/strain type	Stress/strain type
von Mises stress	Stress intensity
Maximum principal stress	Maximum Shear stress
Minimum principal stress	Eqv. plastic strain

Parameter-less Optimization



$$S_{node} = \frac{\sigma_{node} - \sigma_{th}}{\sigma_{max} - \sigma_{min}} \cdot d$$

Coiled Spring Background

- Helical springs are **key components** for many mechanical systems and have been **deeply studied**
- **Stress** distribution is **not uniform**: optimization of the wire is focused on **cross-section** shape
- In this study the BGM driven optimization has been performed considering **two constraints** separately:
  - Cross-section outer radius fixed and **inner surface sculpted**
  - Cross-section inner radius fixed and **outer surface sculpted**
- Optimized geometries have been compared with
  - Equivalent circular cross-section with **same stiffness** and outer/inner **radius**
  - Equivalent circular cross-section with **same stiffness** and swept **volume**

Coiled Spring Background

- Since the analyzed coil **spring** is **flat** and with a spring index > 6, basic spring design formulas can be applied
- **Stiffness**

$$K = \frac{Gd^4}{8D^3}$$

- **Maximum** tangential stress (inner radius)

$$\tau_{in} = \frac{8PD}{\pi d^3}\left(\frac{4c-1}{4c-4} + \frac{0.615}{c}\right)$$

- **Minimum** tangential stress (outer radius)

$$\tau_{out} = \frac{8PD}{\pi d^3}\left(\frac{4c+1}{4c+4} - \frac{0.615}{c}\right)$$

Coiled Spring Background

- In order to meet the **constraints** on outer diameter, inner diameter and volume, the following equation can be applied
- **Same outer diameter and stiffness**

$$D + d = D_e \qquad \frac{Gd^4}{8D^3} = K \longrightarrow Gd^4 - 8(D_e - d)^3 K^* = 0$$

- **Same inner diameter and stiffnes**

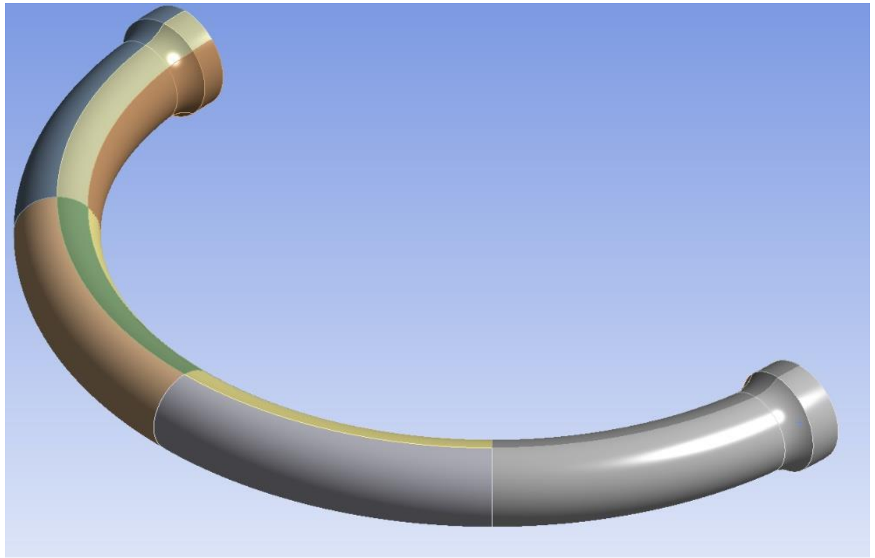
$$D - d = D_i \qquad \frac{Gd^4}{8D^3} = K \longrightarrow Gd^4 - 8(D_i + d)^3 K^* = 0$$

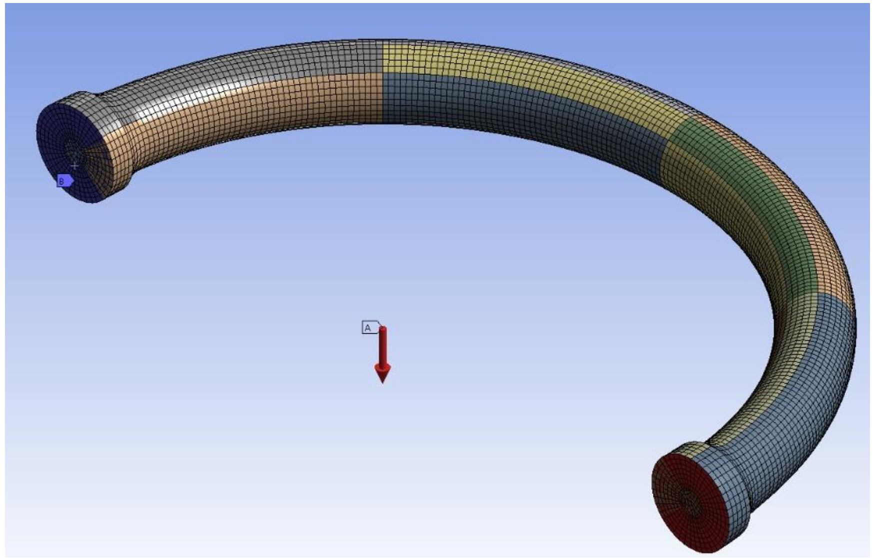
- **Same volume and stiffness**

$$D = \left(\frac{2GV^2}{\pi^4 K}\right)^{\frac{1}{5}} \qquad d = \sqrt{\frac{4V}{\pi^2 D}}$$

Coiled Spring Section Application

- Numerical model represented **half coiled spring**, shaped in order to mitigate stress concentration due to load and constraint application

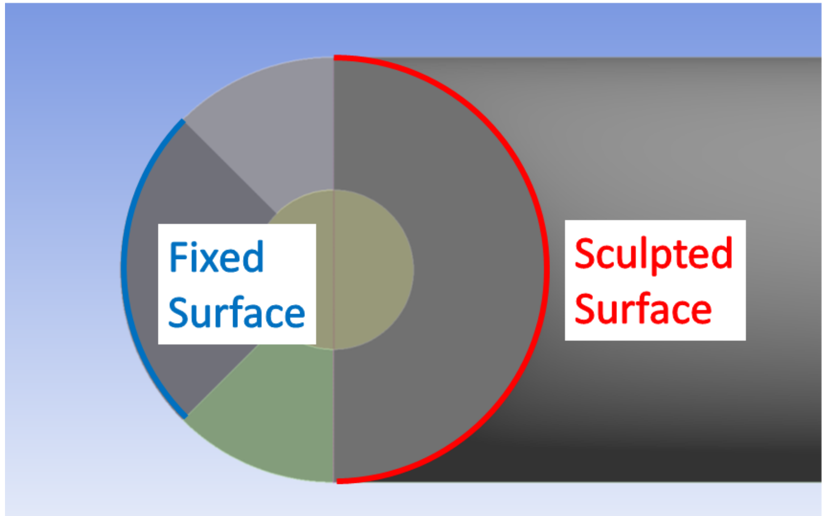


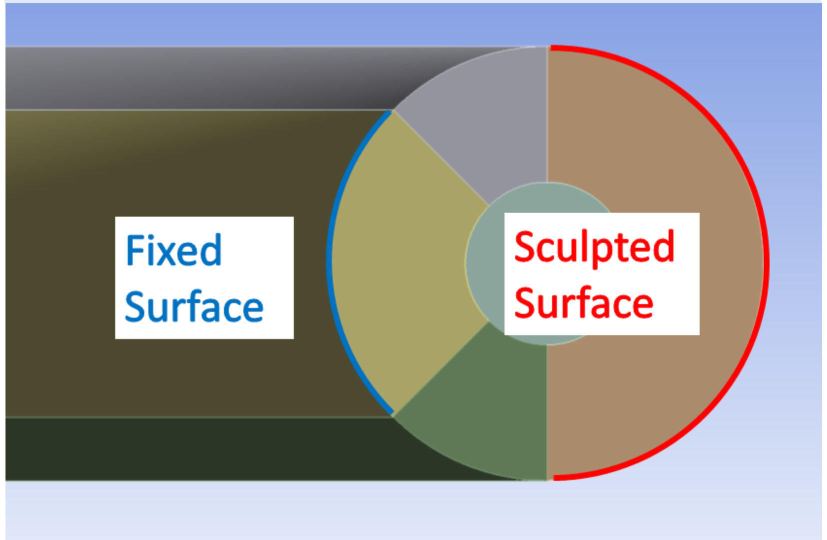


- **74200 parabolic elements** were used to discretize the geometry, resulting in **306’000 nodes**

Coiled Spring Section Application

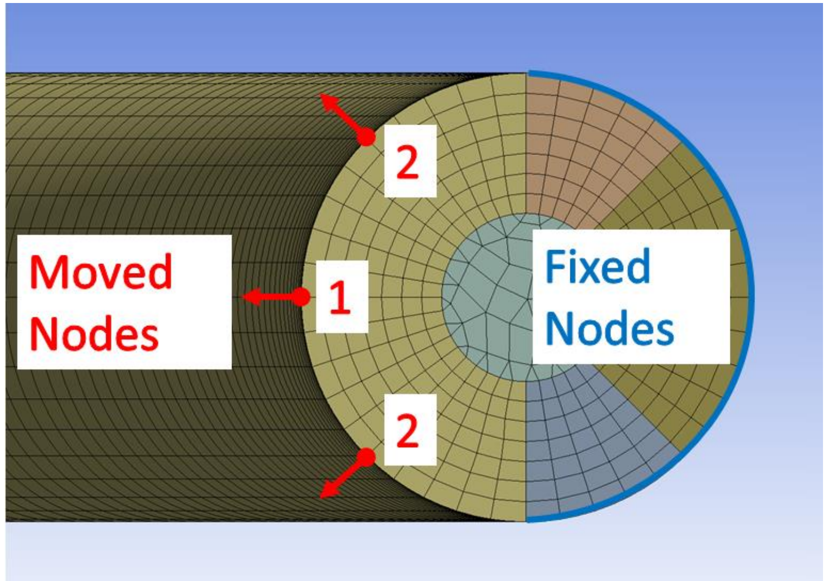
- Geometry was **segmented** in order to ease the surface selection for BGM stress evaluation and morphing action application
- BGM **parameters** *d* and σ_{th} were set equal to 1.2% of wire diameter and 80% of the maximum Equivalent von Mises Stress acting on the coil surface in the baseline configuration
- Optimization was stopped after **20 BGM iterations**





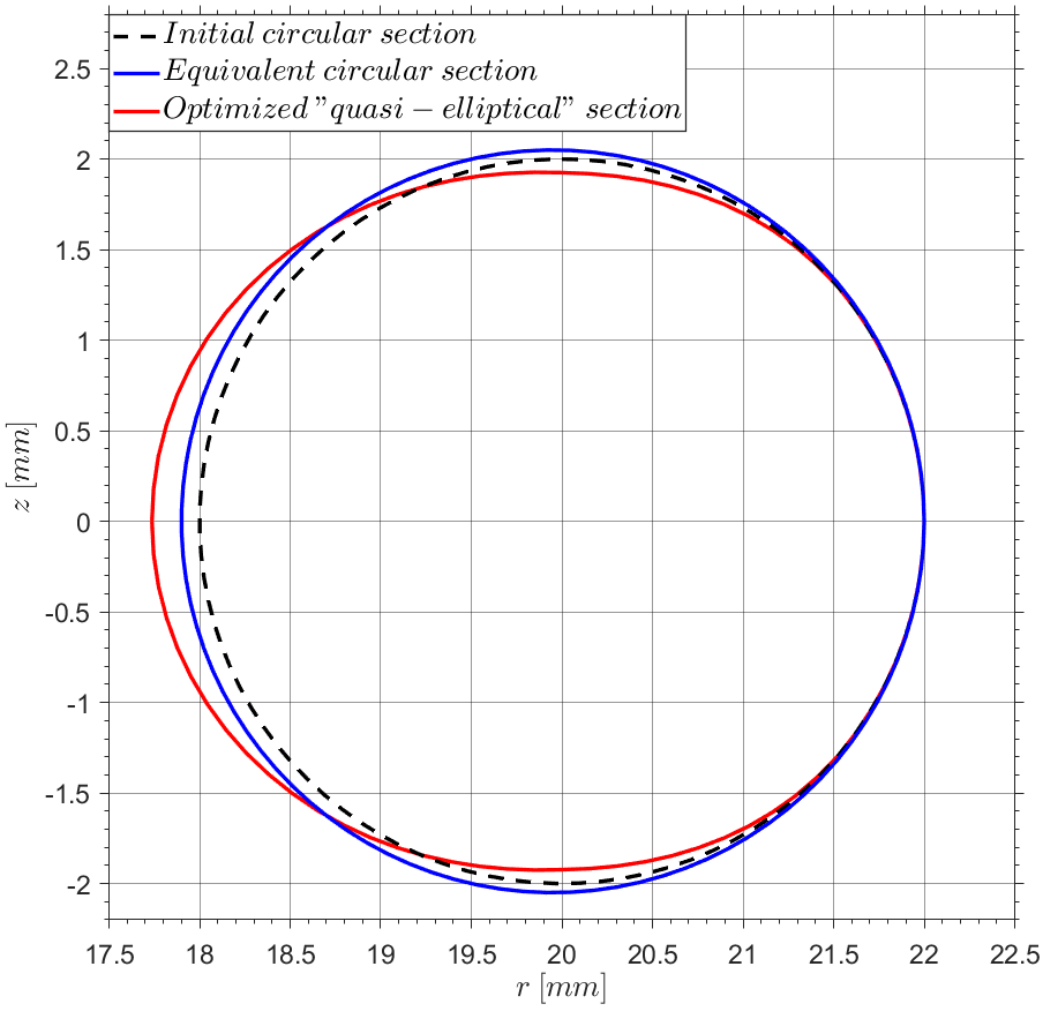
Coiled Spring Section Application

- For comparison purposes, a **parameter-based optimization** was also performed, modifying the inner coil surface using **two parameters**
- Nodes on the inner points and at 45 degrees were **moved along surface normal** in the range [0; 0.2] mm
- ANSYS Design Explorer Response Surface **minimization** was used, generating a Latin Hypercube Sampling Design DoE on which a Kriging response surface with variable kernel was computed



Coiled Spring Section Application

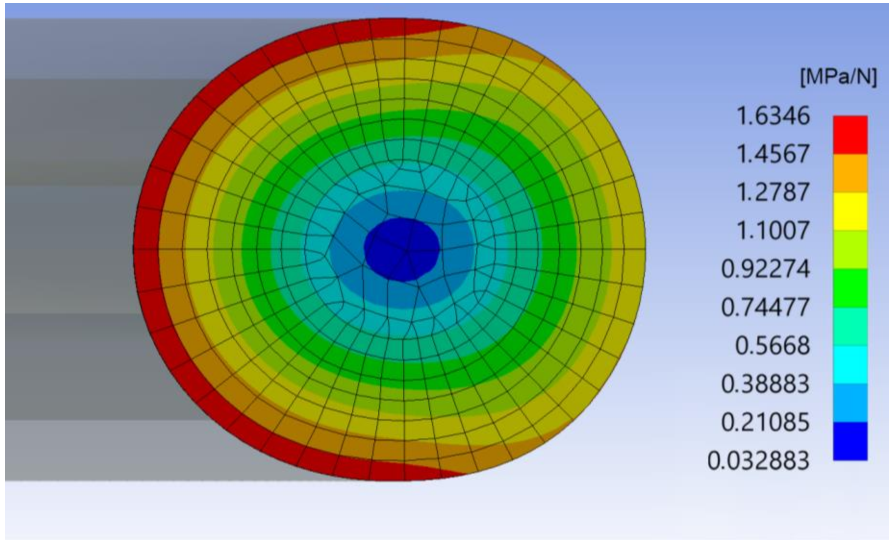
- **Inner surface sculpting**
  - Inner surface is moved outward **adding material**
  - Compared to equivalent circular cross section, **maximum stress is lowered** by **3.73%, increasing volume** by **0.6%**
  - **Efficiency** (ration between elastic energy stored in the section and energy that can be stored with all section points at maximum stress) **is higher** than the equivalent circular section (41% vs 38%)
  - In the **same volume** spring maximum **stress** is 4% **higher** than in the BGM optimized one



Coiled Spring Section Application

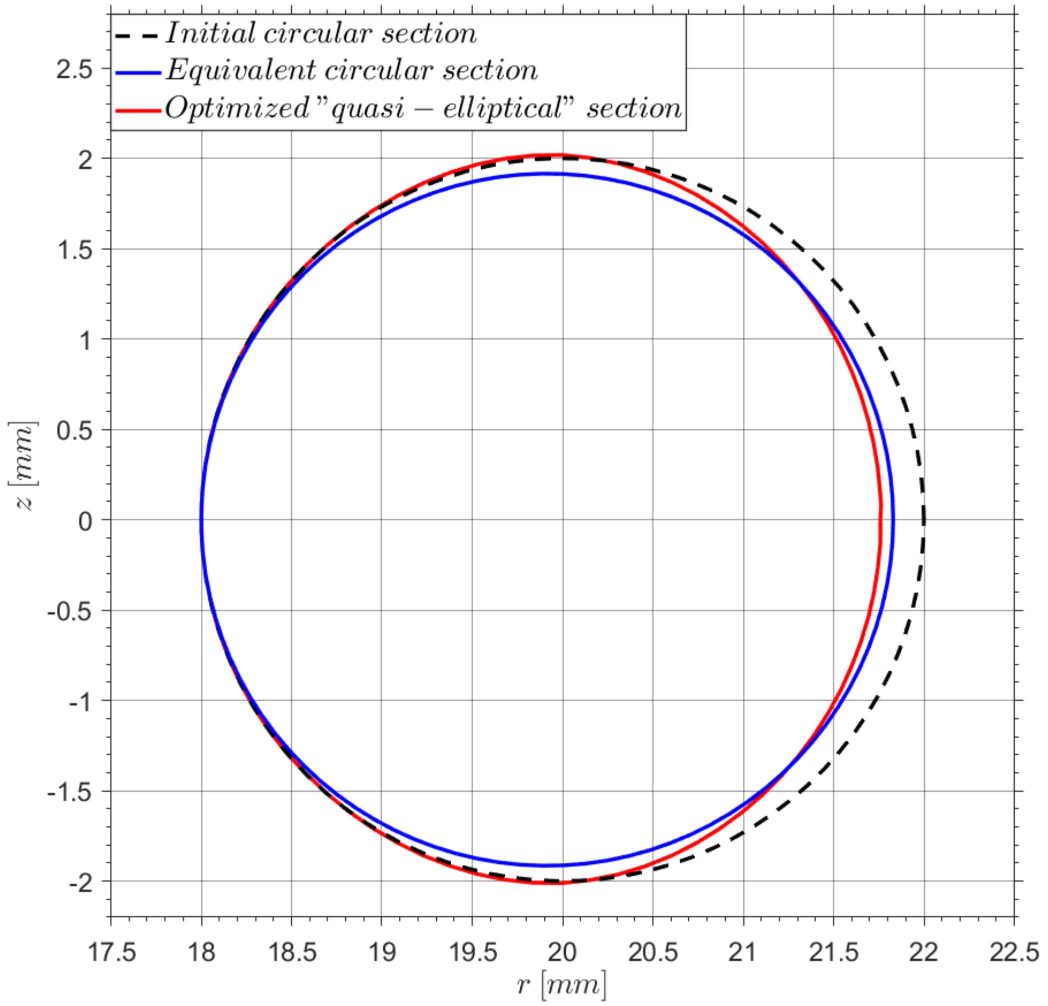
- **Inner surface sculpting**

	d_r [mm]	d_z [mm]	D [mm]	τ_{in}/P [Mpa/N]	τ_{out}/P [Mpa/N]	Eps [-]	A [mm²]	V [mm³]
Optimized	4.26	3.85	39.74	1.635	1.273	41%	13.13	1643
Equivalent	4.10	4.10	39.90	1.696	1.280	38%	13.19	1653
Same volume	4.09	4.09	39.80	1.702	1.515	38%	13.14	1643



Coiled Spring Section Application

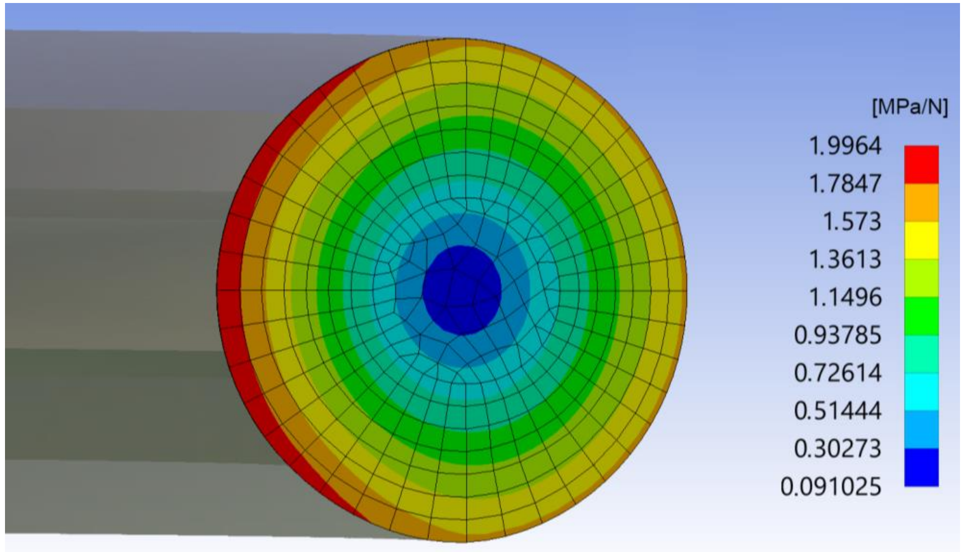
- **Outer surface sculpting**
  - Outer surface is moved inward **removing material**
  - Compared to equivalent circular cross section, **maximum stress is lowered** by **3%**, **increasing volume** by **2.3%**
  - **Efficiency** is **not improved** with respect to the equivalent circular section
  - In the **same volume** spring maximum **stress is 0.5% lower** than in the BGM optimized one



Coiled Spring Section Application

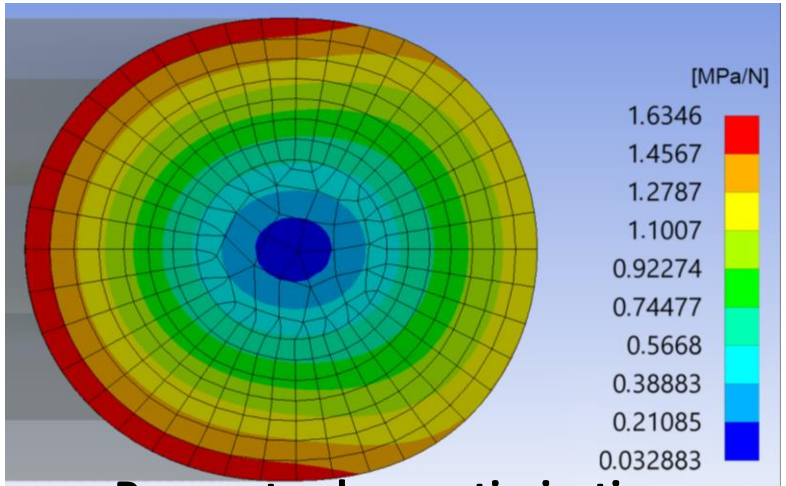
- **Outer surface sculpting**

	d_r [mm]	d_z [mm]	D [mm]	τ_{in}/P [Mpa/N]	τ_{out}/P [Mpa/N]	Eps [-]	A [mm²]	V [mm³]
Optimized	3.76	4.02	39.76	1.996	1.598	38%	11.78	1476
Equivalent	3.83	3.83	39.83	2.055	1.579	38%	11.53	1443
Same volume	3.88	3.88	39.81	1.986	1.777	38%	11.80	1476

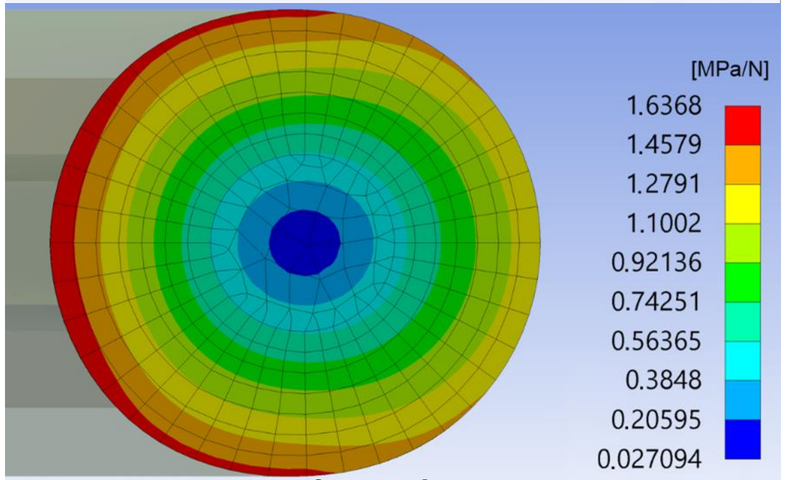


Coiled Spring Section Application

- **Optimization method comparison**
  - With parameter-based optimization **cross-section** area is **1.75% higher**
  - A **higher stiffness** is obtained (+3.76%)
  - **Efficiency** is **38%** (as in equivalent section configurations)
  - **Stress distribution** is **less homogeneous** on surfaces



Parameter-less optimization



Parameter based optimization

Conclusions

- A new **parameter-less** approach for shape optimization has been presented
- **BGM** and **Mesh Morphing** are combined into an innovative surface sculpting tool, capable to take advantage of **surface stress** levels
- **RBF** based **Mesh Morphing** is used to modify shapes according to **BGM** data
- Proposed approach has been applied to a widely investigated mechanical component: **springs**
- The sculpting action was applied to **inner** surface (where **maximum** surface stress acts) and to **outer** surface (where **minimum** surface stress acts)

Conclusions

- BGM sculpting allowed to obtain a cross-section capable to guarantee **higher efficiency** (41%) and **lower maximum** stress (-3.73%) **slightly increasing volume** (+0.6%) when applied to **inner** coil **surface**
- Application to **outer surfaces** did not gave better result if compared to equivalent cross section performances
- A **parameter-based optimization** was compared to the results from parameter-less one: whilst **results are comparable**, spring **efficiency** was **higher** with BGM sculpting optimization
- On the other hand, a parameter-based optimization requires **more user efforts** to complete, whilst BGM optimization can be run **automatically**

Thank You For Your Kind Attention!

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