Geometric Parameterization Strategies for shape Optimization Using RBF Mesh Morphing

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Abstract Mesh morphing is one of the most promising approach for problems in which numerical analyses, based on discretised domains, involve shape parameterization. Some of the benefits associated to its adoption are the reduction of the computational meshing costs and the remeshing noise prevention, guaranteeing at the same time the continuum shape parameterization and consistency of mesh topology. One of the best mathematical tool to drive the mesh morphing (smoothing) task is recognized to be Radial Basis Functions. This paper introduce the RBF Morph tool and lists a set of applications in which the RBF shape parameterization is used to face problems ranging from aerodynamic optimization to Fluid Structure Interaction analyses.

Keywords: Radial Basis Function, mesh morphing, numerical optimization, geometric parameterization.

1 Introduction

The adoption of multidisciplinary numerical optimization (MDO) has become, in the last decade, the standard choice to face most of design problems in the aerospace field. Such methods can be used to study cases ranging from aerodynamic or structural design to dynamic FSI (Fluid Structure Interaction) structural response optimization. When a domain discretization is involved, a critical aspect of MDO based numerical tools, which affects both the efficiency and the quality of the solution, is related to the strategy to be used to implement the geometric parameterization. Most of methods commonly adopted can be divided into two categories: CAD based and mesh morphing based. The first permits to exploit the features of modern parametric CAD systems providing the possibility to manage complex models, great control of the quality of the geometry and large flexibility in variables and constraints definition. The drawback is the necessity to regenerate the computational domain for every new candidate to be investigated introducing uncertainness in the procedure robustness and in the accuracy of mesh depending analysis methods. The remeshing requirements, furthermore, limits the application of automatic CAD based analysis procedures to problems having moderate dimension (in term of computational domain cells number) or to relatively simple geometries suitable to be modelled by structured grids. The mesh morphing approach consists in implementing the geometric parameterization directly on the computational domain using algorithms able to smoothly propagate the model displacement to the surrounding volume.

Several advantages are related to the RBF mesh morphing approach: the robustness of the procedure is preserved, any kind of mesh typology are supported without the need to regenerate it, the smoothing process can be highly parallelizable and can be integrated in any solver. The latter feature offers the very valuable capability to update the computational domain "on the fly" during the progress of the computation. The main disadvantages are the requirement of a "back to CAD" procedure, some limitation in the model displacement amplitude, due to the distortion occurring after morphing, and the high computational cost related to the solution of the RBF system which, if large computational domains are involved, imposes the implementation on HPC environments.

The first commercial mesh morphing software based on Radial Basis Functions was RBF Morph. Its development began in 2008 as a consultancy activity to a top Formula 1 team and continued with the application to typical aerospace engineering problems. Today the software can be fully integrated in several CFD (commercial and open source) and FEM solvers and was successfully used to face many engineering problems that require geometric parameterization (shape optimization, 6DOF analyses, ice accretion, static and dynamic FSI analysis with both 2-ways and modal approach). It was demonstrated that the aforementioned listed disadvantages of mesh morphing shape parameterization were successfully bypassed or practically limited by the efficient implementation of the RBF algorithm. The RBF Morph core technology is at the base of three European research projects, funded within the 7th FP in which the University of Rome "Tor Vergata" is involved. In this paper a description of its working principles and a list of case studies, in which the shape parameterization was implemented using RBF mesh morphing, is presented.

2 Radial Basis Functions

Radial Basis Functions (RBF) are powerful mathematical functions able to interpolate, giving the exact values in the original points, functions defined at discrete points only (source points). The interpolation quality and its behaviour depends on the chosen RBFs. Typical radial functions are reported in Table 1.

Table 1. Typical RBFs.

RBF	$\boldsymbol{\phi(r)}$
Spline type (R_n)	$\lvert r\rvert^n, n$ odd
Thin plate spline (TPS_n)	$\lvert r\rvert^n \log\lvert r\rvert, n$ even
Multiquadric	$\sqrt{1+r^2}$
Inverse multiquadric	$\frac{1}{\sqrt{1+r^2}}$
Inverse quadratic	$\frac{1}{1+r^2}$
Gaussian	e^{-r^2}

A linear system (of order equal to the number of source points introduced) needs to be solved for coefficients calculation [1]. Once the unknown coefficients are calculated, the motion of an arbitrary point inside or outside the domain (interpolation/extrapolation) is expressed as the summation of the radial contribution of each source point (if the point falls inside the influence domain). An interpolation function composed by a radial basis and a polynomial is defined as follows:

$$s(x) = \sum_{i=1}^{N} \gamma_i \, \phi(\|x - x_i\|) + h(x) \tag{1}$$

The minimal degree of polynomial h depends on the choice of the basis function. A unique interpolant exists if the basis function is a conditionally positive definite function. If the basis functions are conditionally positive definite of order $m = 2$, a linear polynomial can be used:

$$h(x) = \beta + \beta_1 x + \beta_2 y + \beta_3 z \tag{2}$$

The values for the coefficients γ of RBF and the coefficients β of the linear polynomial can be obtained by solving the system

$$\begin{pmatrix} M & P \\ P^T & 0 \end{pmatrix} \begin{pmatrix} \gamma \\ \beta \end{pmatrix} = \begin{pmatrix} g \\ 0 \end{pmatrix} \tag{3}$$

where g are the know values at the source points. M is the interpolation matrix defined calculating all the radial interactions between source points

$$M_{ij} = \phi\left(\left\|x_{k_i} - x_{k_j}\right\|\right) \qquad 1 \le i \quad j \le N \tag{4}$$

and P is the constraint matrix

$$P = \begin{pmatrix} 1 & x_{k_1}^0 & y_{k_1}^0 & z_{k_1}^0 \\ 1 & x_{k_2}^0 & y_{k_2}^0 & z_{k_2}^0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{k_N}^0 & y_{k_N}^0 & z_{k_N}^0 \end{pmatrix} \tag{5}$$

The radial basis is a meshless method. Only grid points are moved, regardless of the element connected, and it is suitable for parallel implementation. In fact, once the solution is known and shared in the memory of each calculation node of the cluster, each partition has the ability to smooth its nodes without taking care of what happens outside because the smoother is a global point function and the continuity at interfaces is implicitly guaranteed.

3 RBF Morph description

RBF Morph is a numerical suite for morphing and shape optimization that combines a very accurate control of the geometrical parameters with an extremely fast mesh deformation capability. The tool was born as an add-on of the ANSYS Fluent CFD code and is fully integrated in the solving process [2]. Today RBF Morph is also available as a standalone library to be coupled with any code. It was successfully embedded in the solving process with OpenFOAM, CFD++, elsA, StarCCM+ and the FEM solvers NASTRAN and ANSYS Mechanical.

The industrial implementation of the RBF mesh morphing poses two challenges: the numerical complexity related to the solution of the RBF problem for a large number of centres and the definition of suitable paradigms to effectively control shapes using RBF. The RBF Morph software allows to deal with both as it comes with a fast RBF solver capable to fit large dataset (hundreds of thousands RBF points can be fitted in few minutes) and with a suite of modelling tools that allow the user to setup each shape modification in an expressive an flexible way.

RBF Morph allows to extract and control points from surfaces and edges, to put points on primitive shapes (boxes, spheres and cylinders) or to specify them directly by individual coordinates and displacements. Primitive shapes can be combined in a Boolean fashion allowing to limit the action of the morpher itself. The shape information coming from an individual RBF setup are generated interactively with the help of the GUI and are used subsequently in batch commands that allow to combine many shape modifications in a non-linear fashion (non linearity occurs when rotation axis are present in the RBF setup). The displacement of the prescribed set of source points can be amplified according to parameters that constitutes the parametric space of the shape model.

The most important features of the RBF mesh morphing are: it provides a mesh-independent solutions; the morphing action can be highly parallelizable; very large models (hundreds of millions of cells) can be morphed in few minutes and every kind of mesh element type (tetrahedral, hexahedral, polyhedral, prismatic, non-conformal interfaces, etc.) are supported. Fig. 1 reports an example of an RBF mesh morphing action applied to the analysis of a motorbike windshield.

Fig. 1. Source points of an RBF problem and result of the mesh morphing action.

Mesh morphing with RBF Morph is executed in three steps:

1. definition and setup of the problem;
2. solution of the RBF system (fitting);
3. morphing of surface and volume meshes (smoothing).

The smoothing action is performed firstly applying the prescribed displacement to the grid surfaces and then smoothly propagating the deformation to the surrounding domain volume. A back2CAD feature is implemented in order to generate a CAD model of the modified geometry. The principle is to apply the RBF setup to the source CAD model in STEP format (CAD morphing). The method is not based on a total CAD regeneration but rather on a synchronization of the CAD surfaces and the mesh. The method is not exact but in our experience the discrepancies between the mesh and the generated CAD is in general very small.

4 Application of RBF

Several engineering problems can be efficiently faced by mesh morphing. Examples of applications in several fields can be found in [3] ,[4], [5] and [6]. Aerodynamic optimization is easily implemented by very flexible shape parameterization capabilities. In [7] a car shape optimization coupled to an Adjoint method, using the OpenFOAM CFD solver, is reported (Fig. 2).

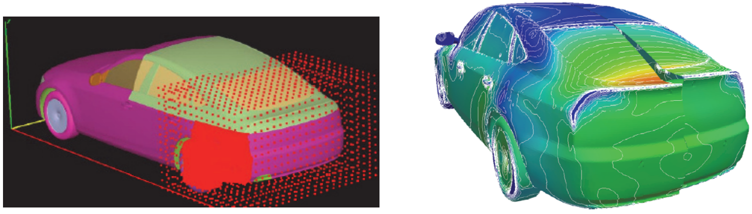

Fig. 2. Mesh morphing for car shape optimization.

RBF shape optimizations applied to the aerodynamic performance improvement of a glider in manoeuvring is reported in [8] (Fig. 3). Other examples of aerodynamic optimization problems faced with RBF Morph in aeronautics are reported in [9] and [10].

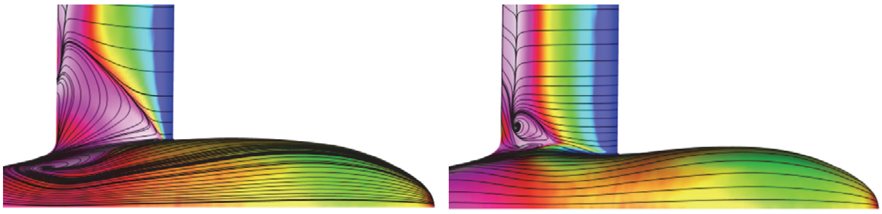

Fig. 3. Wing/fuselage interference optimization of a glider

Fluid Structure Interaction (FSI) problems are very efficiently approached coupling the structural to the fluid dynamic solution by mesh morphing. The classical approach, also called CFD-CSM (Computational Fluid Dynamic – Computational Structural Mechanics) or 2-ways, consists in iterating between CFD and FEM solver according to the scheme reported in Fig. 4 [11]. In such procedure the pressure from the CFD solution has to be mapped into the FEM grid as loads. The interpolation between the two non-conformal domains is also performed applying RBF.

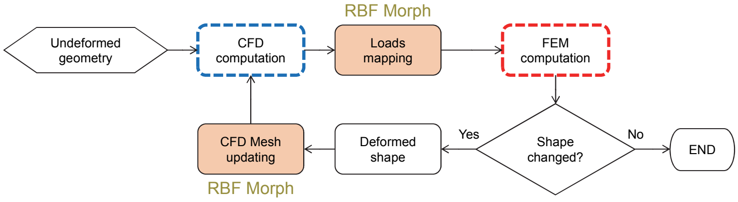

Fig. 4. 2-ways FSI procedure

RBF mesh morphing is suitable to face steady [12] and unsteady [13] FSI analyses by a modal approach. The principle consists in setting up a database of RBF solutions of a number of structural natural modal shapes to be amplified, according to the modal coordinates computed and updated during the CFD iterations, and combined to replicate the structural deformation under loads. The CFD environment becomes, with this approach, an intrinsically aeroelastic analysis that do not involves any further structural analysis. The flow chart of the modal FSI analysis setup is reported in Fig. 5 together with the examples of the four first modal shapes of a wing and the formulations used to render the mesh parametric.

Another very challenging task for mesh morphers is the ice accretion problem on aircrafts. RBF Morph demonstrated the capability to robustly replicate very complex ice shapes maintaining acceptable mesh quality [14].

Computational costs of the RBF morphing action is known to be a critical aspects. Thanks to its high parallelizability, very large problems can be faced on

HPC environments. To the authors knowledge, the largest model managed with RBF Morph has 700 millions of cells and was morphed in 45 minutes using 768 CPUs. Table 2 reports three samples of the solver performance detailing the fitting and the smoothing action elapsed time.

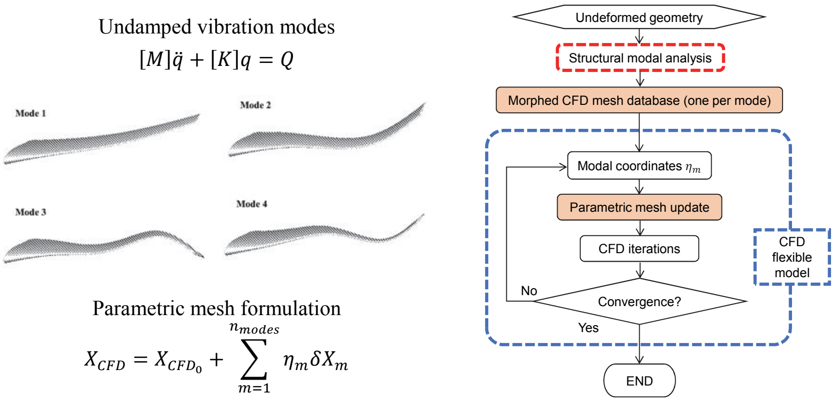

Fig. 5. Modal approach for FSI analysis

Table 2. RBF solver performance samples.

Mesh cells	*Source points*	*CPUs*	*Fitting time(serial)*	*Smoothing (parallel)*
14 mill.	60.000	4	53 sec.	3.5 min
50 mill.	30.000	140	25 sec.	1.5 min.
100 mill.	200.000	256	25 min	5 min.

5 Conclusions

A geometric parameterization method, suitable to be adopted in numerical analyses that require volumes or surfaces domains discretization, was presented. The method is based on mesh morphing approach using Radial Basis Functions. RBF Morph was the first commercial mesh morphing software based on RBFs. The qualities and the performance of the tool was demonstrated by reporting a set of engineering application ranging from shape optimizations to FSI analyses. The high parallelizability of the RBF solver provides, furthermore, the capability to manage very large mesh morphing problems. The workflow can be easily and efficiently automated and coupled to any flow or structural solver. The quality of its morphing action was demonstrated also on very challenging problems as aircrafts ice accretion. In comparison to a CAD and remesh driven, the RBF mesh morphing approach has the advantage to reduce the time to setup, can be applied to any type of grids, it prevents the remeshing noise and maintains high robustness levels of the process.

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